Esercizio P2.13

Un vaso troncoconico con sommità cilindrica pieno d'acqua ha le dimensioni indicate in figura. Determinare il modulo S della spinta sulla sola superficie troncoconica.

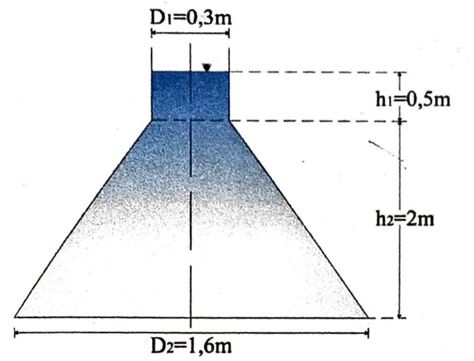
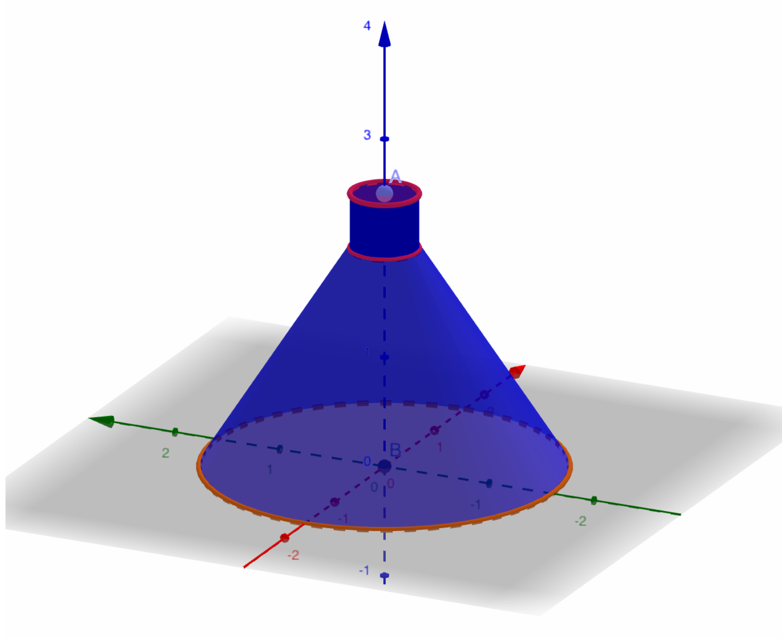


Figura P2.13

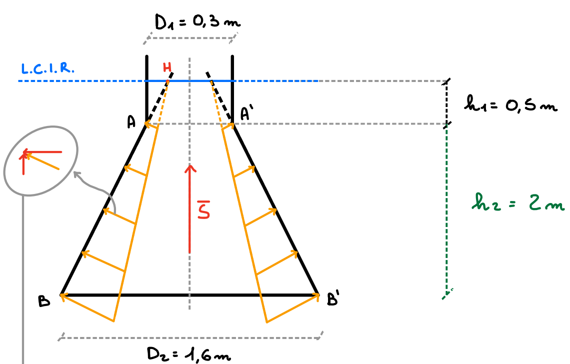


DATI

$D_1 = 0{,}3\,m \qquad D_2 = 1{,}6\,m$

$h_1 = 0{,}5\,m \qquad h_1 = 2\,m$

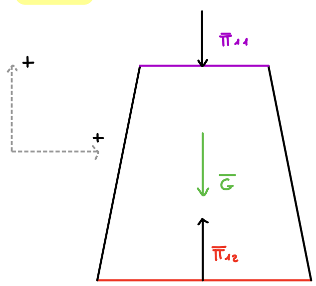
$\gamma = 9800\,\frac{N}{m^3}$

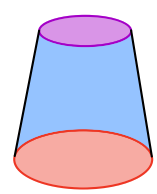


$P_H = \gamma \cdot h = 0\,Pa$

$P_B = \gamma h = 9800 \cdot (h_1 + h_2) = 9800 \cdot 2{,}5 = 24500\,Pa$

CASO 1:





W TRONCO CONICO : $\frac{\pi h}{3} \cdot (R^2 + r^2 + Rr)$

I VETTORI DELLA PRESSIONE DAL PUNTO DI VISTA LOCALE HANNO COMPONENTI VERTICALI e ORIZZONTALI.
DAL PUNTO DI VISTA GLOBALE, INTEGRANDO LE PRESSIONI PER CALCOLARE LA SPINTA LE COMPONENTI ORIZZONTALI SI ELIDONO PERCHÉ SONO UGUALI ED OPPOSTE DATA LA SIMMETRIA DELLA FIGURA. QUINDI RIMANGONO SOLO LE COMPONENTI VERTICALI, CIÓ IMPLICA CHE COMPLESSIVAMENTE LA SPINTA È SOLO VERTICALE.

$$S = \bar{\Pi}_1 + \bar{G} = \bar{\Pi}_{11} + \bar{\Pi}_{12} + \bar{G}$$

$$|\bar{\Pi}_{11}| = \gamma h_0 A = 9800 \cdot h_1 \cdot \pi \left(\frac{D_1}{2}\right)^2 = 346{,}3\,N$$

$$|\bar{\Pi}_{12}| = \gamma h_0 A = 9800 \cdot (h_1 + h_2) \cdot \pi \left(\frac{D_2}{2}\right)^2 = 49200\,N$$

$$|\bar{G}| = \gamma W = 9800 \cdot \frac{\pi h_2}{3} \cdot \left[\left(\frac{D_2}{2}\right)^2 + \left(\frac{D_1}{2}\right)^2 + \left(\frac{D_2}{2} \cdot \frac{D_1}{2}\right)\right] = 16061\,N$$

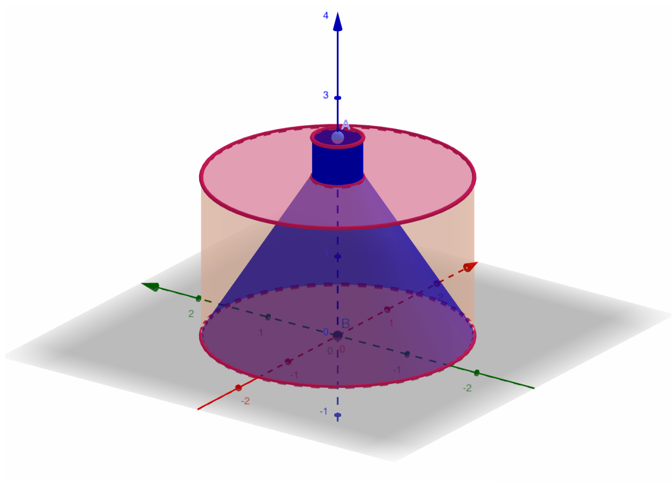
$$S_0 = 0\,N$$

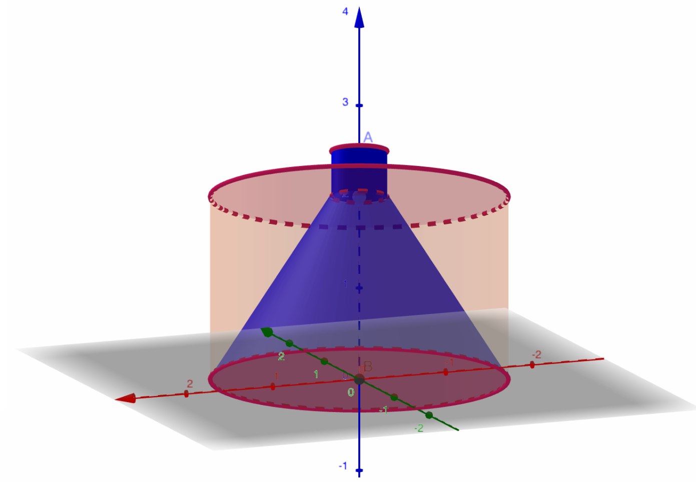
$$S_v = -|\bar{\Pi}_{11}| + |\bar{\Pi}_{12}| - |\bar{G}| =$$

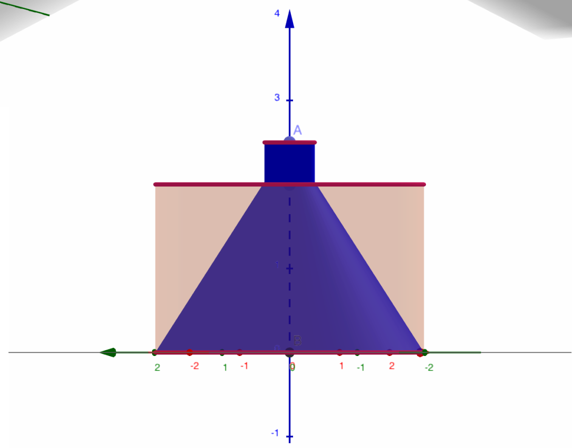
$$= -346{,}3 + 49200 - 16061 = 32852{,}6\,N$$

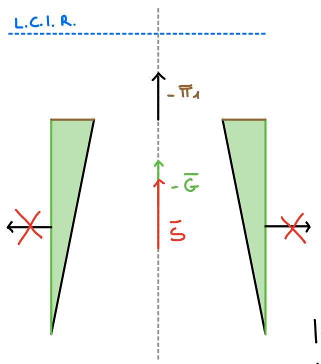
$$S = \sqrt{S_0^2 + S_v^2} = 32852{,}6\,N$$

CASO 2 CON CILINDRO:





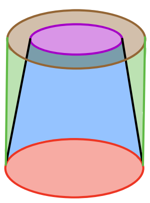




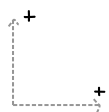
$$S = -\overline{\Pi}_1 - \overline{G}$$

$$|-\overline{\Pi}_1| = \gamma h_0 A = 9800 \cdot h_1 \cdot \left(\pi\left(\frac{D_2}{2}\right)^2 - \pi\left(\frac{D_1}{2}\right)^2\right) = 9505,6 \text{ N}$$

$$|-\overline{G}| = \gamma W = 9800 \cdot (W_{CIL} - W_{\substack{TRONC. \\ CONO}}) = 9800 \cdot \left\{\left(\pi\left(\frac{D_2}{2}\right)^2 \cdot L\right) - \left[\frac{\pi h_2}{3} \cdot \left(\left(\frac{D_2}{2}\right)^2 + \left(\frac{D_1}{2}\right)^2 + \left(\frac{D_2}{2} \cdot \frac{D_1}{2}\right)\right)\right]\right\} = 23346,8 \text{ N}$$



CILINDRO

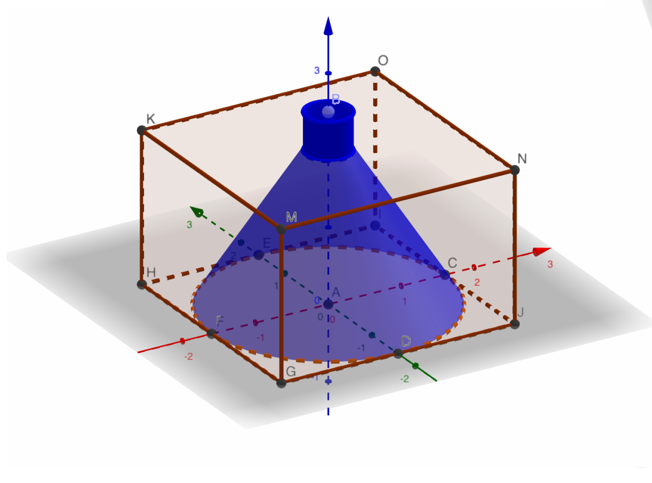


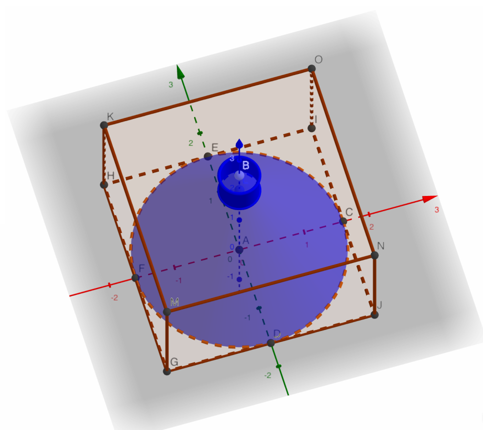
$$S_o = 0 \text{ N}$$

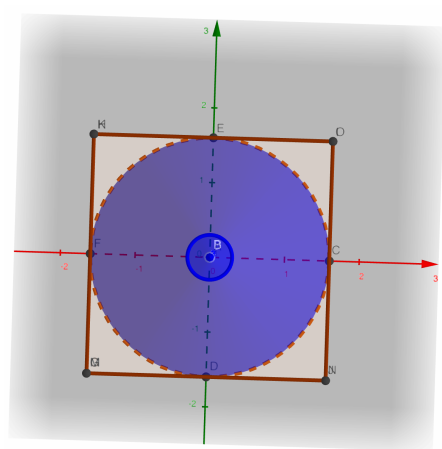
$$S_v = +|-\overline{G}| + |-\overline{\Pi}_1| = 23346,8 + 9505,6 = 32852,4 \text{ N}$$

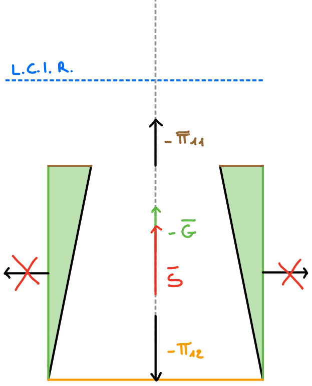
$$S = \sqrt{S_o^2 + S_v^2} = 32852,4 \text{ N}$$

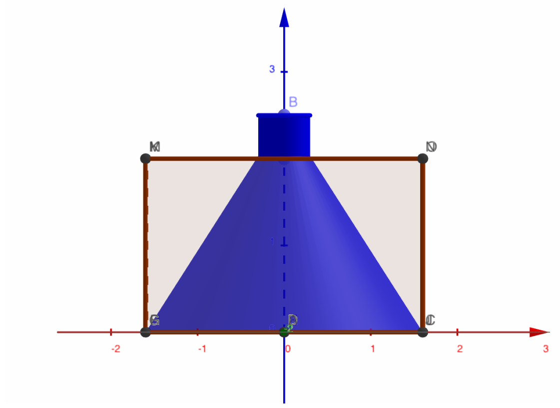
CASO 2 CON PARALLELEPIPEDO:

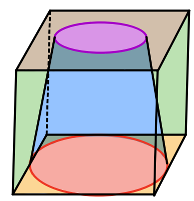












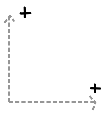
PARALLELEPIPEDO

$$S = -\overline{\Pi}_1 - \overline{G} = -\overline{\Pi}_{11} - \overline{\Pi}_{12} - \overline{G}$$

$$|-\overline{\Pi}_{11}| = \gamma h_o A = 9800 \cdot h_1 \cdot \left(D_2^2 - \pi\left(\frac{D_1}{2}\right)^2 \right) = 12\,197{,}6 \text{ N}$$

$$|-\overline{\Pi}_{12}| = \gamma h_o A = 9800 \cdot (h_1 + h_2) \cdot \left(D_2^2 - \pi\left(\frac{D_2}{2}\right)^2 \right) = 13\,459{,}8 \text{ N}$$

$$|-\overline{G}| = \gamma W = 9800 \cdot (W_{\text{paral.}} - W_{\text{tronc. cono}}) = 9800 \cdot \left\{ \left(D_2^2 \cdot h_2 \right) - \left[\frac{\pi h_2}{3} \cdot \left(\left(\frac{D_2}{2}\right)^2 + \left(\frac{D_1}{2}\right)^2 + \left(\frac{D_2}{2} \cdot \frac{D_1}{2}\right) \right) \right] \right\} = 34\,115{,}1 \text{ N}$$



$$S_o = 0 \text{ N}$$

$$S_v = + |-\overline{G}| + |-\overline{\Pi}_{11}| - |-\overline{\Pi}_{12}| = 34\,115{,}1 + 12\,197{,}6 - 13\,459{,}8 = 32\,852{,}9$$

$$S = \sqrt{S_o^2 + S_v^2} = 32\,852{,}9 \text{ N}$$

Pierpaolo Spinelli - Corso di laurea ingegneria Civile e Ambientale anno 2022/2023