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EXPERIMENTAL STUDY ON THE SCOUR DOWNSTREAM OF GRADE-CONTROL STRUCTURES

M. MOSSA

Dipartimento di Ingegneria delle Acque, Politecnico di Bari, Bari, Italy

ABSTRACT

The problem of scour downstream of hydraulic structures is of great importance. Grade-control structures prevent from excessive channel-bed degradation in alluvional channel. The flowing of water, however, is generally an erosive action that causes significant downstream local scour, which has to be well studied, because it can undermine these structures. The present study deals experimentally with the scour downstream of a grade control structure to get better understanding of the problem, in order to develop a safe design and proper protection works. The maximum equilibrium scour depths measured are compared with those of some literature formulae. Generally these formulae are not dimensionally homogeneous. Therefore, a dimensional analysis is performed to better estimate the results of the present study.

SOMMARIO

Il problema dell'escavazione a valle di strutture di controllo tipicamente usate nel campo delle costruzioni idrauliche è di grande importanza. Le strutture di controllo a gradino vengono utilizzate al fine di prevenire fenomeni di erosione eccessiva dei letti nei canali in cui i getti affluiscono. Ciononostante, l'azione della corrente è erosiva e provoca escavazioni localizzate a valle delle strutture di controllo. La presente memoria riporta i risultati sperimentali delle escavazioni a valle delle strutture di controllo a gradino. Lo studio ha consentito di confrontare i risultati sperimentali delle massime escavazioni di equilibrio con i risultati di alcune formule riportate in letteratura. Tali formule spesso non sono dimensionalmente corrette. Pertanto si presenta un'analisi dimensionale del fenomeno che ben riesce a interpretare i risultati sperimentali del presente studio.

1. INTRODUCTION

Grade-control structures are considered to be one of the most important ways to prevent excessive degradation in alluvial channels. The erosive action of the flowing water causes a significant downstream local scour, which may cause the instability of the structure.

Many laboratory investigations on the scour depths under various flow conditions and structure configurations are available. Mason and Arumugam (1985) observed that there are many formulae which have been developed for estimating scour under a falling. A list of these formulae is reported by Whittaker and Schleiss (1983) and Mason and Arumugam (1985). All these formulae were obtained by experimental results.

The scour downstream of a grade-control structure was studied by Bormann and Julien (1991). The authors defined an equilibrium scour equation based on the concept of the jet diffusion and particle stability in scour hole of grade-control structure and tested it comparing the results with large-scale experiments.

Taking into account the complexity of such problem, the present study will deal with the scour downstream of grade-control structure providing different case studies of such phenomena for several configurations of wall jets. In order to investigate the scouring process features, several experimental tests were carried out with two different configurations of the grade-control structure. The experimental data were compared with those calculated using some literature formulae, that generally are not dimensionally homogeneous. A dimensional analysis of the scour phenomena is proposed and the experimental results of the present study are compared with the proposed formula.

2. EXPERIMENTAL SET-UP

The tests concerning the investigation of the scouring process for the current case study were carried out at the hydraulic laboratory of the Mediterranean Agronomic Institute of Bari (Italy), in a 7.72 m long channel made of Plexiglas floor and glass lateral walls, having a rectangular section of 0.30 m width by 0.40 m depth. Water is fed from an upstream reservoir with a maximum charge of 54 cm, equipped with stilling grid and lateral weir, that maintains a constant head. At the downstream end of the flume, a stilling chamber is installed and provided with a triangular weir to measure the channel discharge. Water discharge and hydrodynamic conditions were regulated by two gates placed upstream and downstream of the channel. The desired tailwater level was maintained by using the downstream gate. The experimental layout is shown in Figs. 1 and 2.

The experimental layout mainly consists of three wooden models of the structure. The downstream part of the wooden model was welded to the flume walls to form two structure face slopes tested. The upstream face of the structure models is sloped properly to create a smooth transition from the upstream reservoir to the flume.

The flume bottom after the structure model is covered with an erodible bed material consisting of sand particles of almost uniform size with grain diameter corresponding to 50% finer d_{50} equal to 2.0 mm and specific gravity equal to 2.65. The thickness of the sand layer is 0.16 m. Another small wooden part is located at the end of the bed material layer.

The profile of the eroded bed was measured by means of a point gauge supplied with a vernier which allowed measurements with accuracy of ±0.1 mm. Water depths were measured with hydrometers supplied with electronic integrators which allowed estimation of the time-averaged depth of the flow with accuracy of ±0.1 mm.

3. SOME LITERATURE SCOUR FORMULAE

As proposed by Mason and Arumugam (1985) the equilibrium scour depth equations in the power form is

$$D_s + Y_t = K q^a U_0^b \Delta H^c Y_t^d \beta^{ie} g^{-f} d_s^{-i} \quad (1)$$

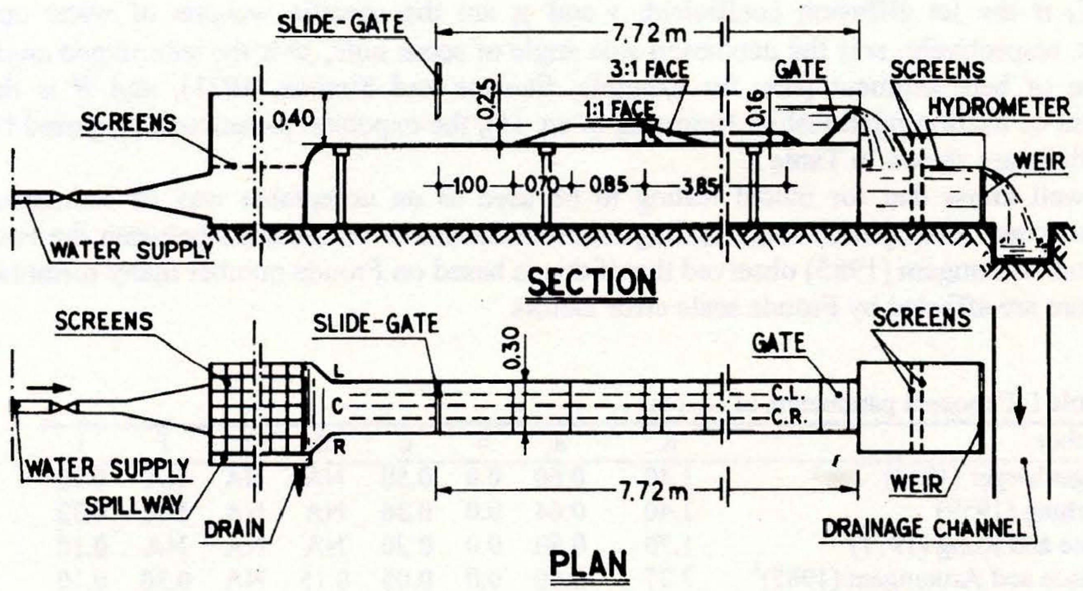


Fig. 1. Schematic view of experimental arrangement

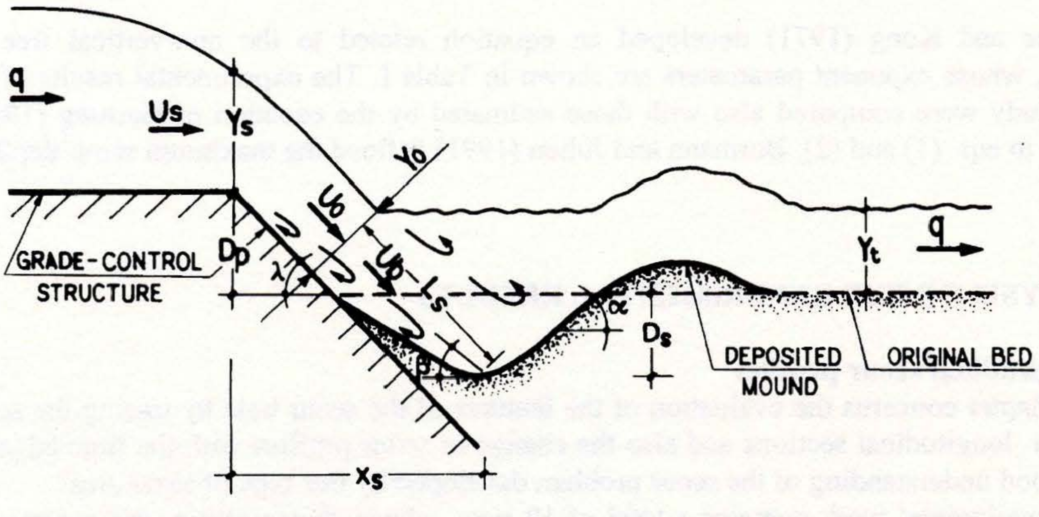


Fig. 2. Definition sketch of the jet path

where *a*, *b*, *c*, *d*, *e*, *f*, and *i* are the exponents of the scour equation; D_s is the equilibrium scour depth (m), Y_t is the tailwater depth above unscoured bed level (m), q is the unit water discharge (m^2/s), U_0 is the jet velocity entering the tailwater (m/s), ΔH is the head drop across the structure (m), β' is the jet angle near bed (in radiants, but in some formulae sin β' is present), g is the gravitational acceleration (m/s^2), d_s is the effective sediment size (m), and K is the constant in scour equation. Bormann and Julien (1991) observed that K depends on the jet configuration and proposed the following equation

$$K = C_d^2 \left[\gamma \sin\phi / \left(\sin(\phi + \alpha) B(\gamma_s - \gamma)\right)\right]^{0.8} \tag{2}$$

where C_d is the jet diffusion coefficient, γ and γ_s are the specific weights of water and sediment, respectively, α is the maximum side angle of scour hole, Φ is the submerged angle of repose of bed sediment (see, for example, Stevens and Simons, 1971), and B is the coefficient of friction relationship. Referring to eq. (1), the exponent parameters proposed by some authors are shown in Table I.

It is well know that for model testing to be used as an acceptable way of estimating prototype scour development some scaling relationship has to be assumed between the two. Mason and Arumugam (1985) observed that if this is based on Froude number many formulae in literature are affected by Froude scale error factors.

Table I. Exponent parameters of eq. (1)

Author	K	a	b	c	d	e	f	i
Eggenberger (1943)	1.40	0.60	0.0	0.50	NA	NA	NA	0.40
Hartung (1959)	1.40	0.64	0.0	0.36	NA	NA	NA	0.32
Chee and Kung (1971)	1.70	0.60	0.0	0.20	NA	NA	NA	0.10
Mason and Arumugam (1985)[1]	3.27	0.60	0.0	0.05	0.15	NA	0.30	0.10
Bormann and Julien (1991)	Eq. (2)	0.60	1.0	NA	NA	1.0[2]	0.8	0.40

[1] Summary of many previous equations, values of exponent vary with the head.
[2] Uses sin β'.
Note: NA = not applicable.

Chee and Kung (1971) developed an equation related to the non-vertical free jet condition, whose exponent parameters are shown in Table I. The experimental results of the present study were compared also with those estimated by the equation of Hartung (1959). Referring to eqs. (1) and (2), Bormann and Julien (1991) defined the maximum scour depth of 88 runs.

4. ANALYSIS OF THE EXPERIMENTAL RESULTS

4.1. Longitudinal scour profiles

This chapter concerns the evaluation of the features of the scour hole by tracing the scour profiles in longitudinal sections and also the change in scour profiles with the time advance to get a good understanding of the scour problem developed by this type of structures.

The experimental work contains a total of 19 runs, whose characteristics are reported in Table II, where Y_s is the flow depth on the sill end, U_s is the mean flow velocity on the sill end, Fr_s is the Froude number of the flow on the sill end, U_{Yt} is the downstream mean flow velocity, Fr_{Yt} is the downstream Froude number and X_s is the distance of the maximum scour point from the sill section. Ten experiments refer to the wall jet case with downstream face slope of 1:1 (face angle of the structure λ=45°) and nine experiments to the wall jet case with downstream face slope of 3H:1V (λ roughly equal to 18°). Through the obtained experimental results, a description of the scouring phenomena can be given in terms of the items described below.

For the analyzed configurations, the equilibrium longitudinal profiles of the scour holes downstream of the structure were traced along five longitudinal sections, shown in Fig. 1. These sections are called R (the right wall of the channel), CR (in the middle between the right wall and the center longitudinal profile), C (center longitudinal profile), CL (in the middle between the left wall and the center longitudinal profile) and L (left wall). For the sake of brevity, the next figures report the results of only three runs for each experimental set-up.

Table II. Summary of experimental data

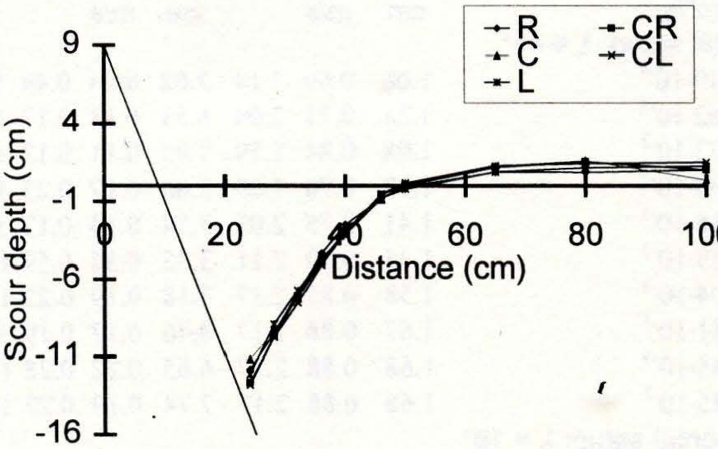
Q m^3/s	Y_s cm	U_s m/s	Fr_s	Y_t cm	U_{Yt} m/s	Fr_{Yt}	D_s cm	X_s cm
First experimental setup: $\lambda = 45°$								
$2.19 \cdot 10^{-3}$	1.06	0.69	2.14	3.02	0.24	0.44	9.00	19
$2.62 \cdot 10^{-3}$	1.23	0.71	2.04	6.53	0.13	0.17	8.20	19
$2.73 \cdot 10^{-3}$	1.08	0.84	2.59	7.95	0.11	0.13	6.00	17
$2.90 \cdot 10^{-3}$	1.38	0.70	1.90	5.66	0.17	0.23	9.60	20
$3.18 \cdot 10^{-3}$	1.41	0.75	2.02	7.34	0.14	0.17	9.25	20
$3.39 \cdot 10^{-3}$	1.43	0.79	2.11	3.35	0.34	0.59	12.50	25
$4.04 \cdot 10^{-3}$	1.58	0.85	2.17	7.18	0.19	0.22	11.45	24
$4.41 \cdot 10^{-3}$	1.67	0.88	2.17	8.46	0.17	0.19	9.80	21
$4.45 \cdot 10^{-3}$	1.68	0.88	2.17	6.65	0.22	0.28	12.60	23
$4.45 \cdot 10^{-3}$	1.68	0.88	2.17	7.74	0.19	0.22	10.90	21
Second experimental setup: $\lambda = 18°$								
$1.36 \cdot 10^{-3}$	0.95	0.48	1.56	2.54	0.18	0.36	3.52	38
$2.62 \cdot 10^{-3}$	1.44	0.60	1.61	3.01	0.29	0.53	8.80	54
$2.95 \cdot 10^{-3}$	1.57	0.62	1.70	8.10	0.12	0.14	5.00	43
$3.28 \cdot 10^{-3}$	1.64	0.66	1.55	3.38	0.32	0.56	10.50	56
$3.52 \cdot 10^{-3}$	1.75	0.67	1.77	6.05	0.19	0.25	7.50	47
$3.88 \cdot 10^{-3}$	1.81	0.71	1.60	3.53	0.37	0.62	13.65	67
$4.07 \cdot 10^{-3}$	1.98	0.68	1.66	8.59	0.16	0.17	6.20	46
$4.22 \cdot 10^{-3}$	1.86	0.75	1.62	7.85	0.18	0.20	8.60	47
$4.32 \cdot 10^{-3}$	1.97	0.73	1.66	6.23	0.23	0.29	14.50	59

Figures 3 a, b and c show the equilibrium longitudinal profiles of the wall jet case with 1:1 downstream face slope versus the distance from the sill end. The figures show that the flow and the scour holes can be assumed two-dimensional, since the wall effect is generally so small to be neglected. The figures show a quite similar shape of the scour profiles with a shift of the position of the maximum scour that depends on the hydrodynamic condition established in the channel. Analogous results refer to the case of 3:1 downstream face slope, which are shown in Figs. 4 a, b and c. In all cases of the present study it was deserved that both the depths of the scour hole and the heights of the deposition part increases with the increase of discharge if the tailwater depth is roughly constant. Besides, under the condition of roughly constant discharge, as the tailwater depth increases, a reduction in the length of the deposit and a growth of the angle of the downstream edge of the scour hole were highlighted.

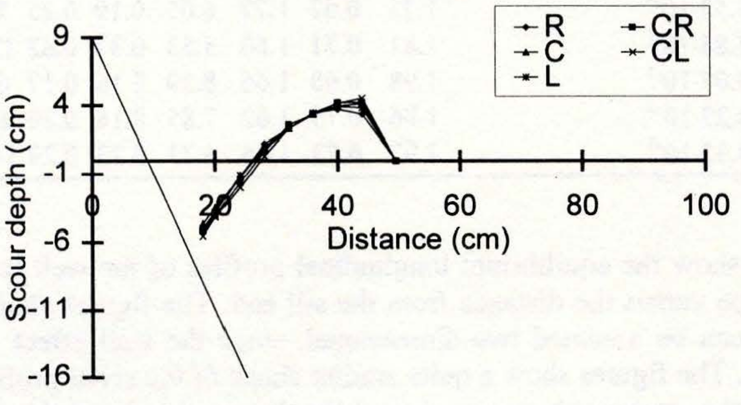
An initial tailwater depth affects the value of the maximum scour depth through an inversely proportional manner, as shown in the previous figures. The tailwater depths change the configuration of the jet diffusion under the condition of a constant discharge and fixed height of the structure model.

4.2. Developments of the scour hole profiles with time advance

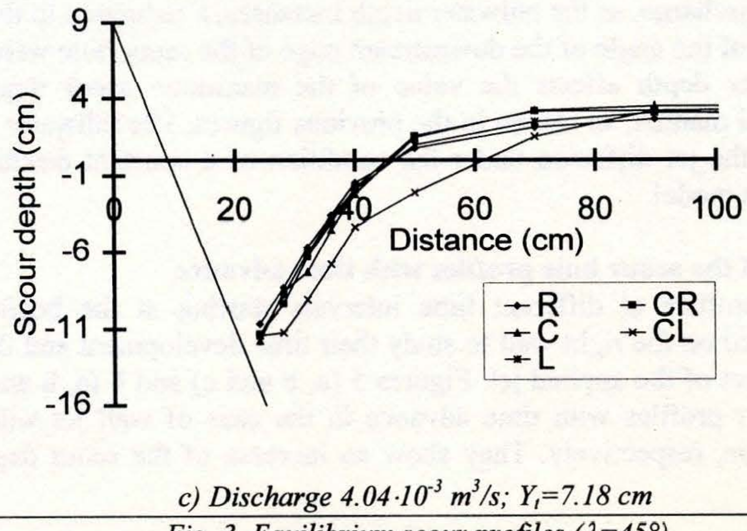
The scour hole profiles at different time intervals starting at the beginning of each experiment were traced on the right wall to study their time development and the geometrical change under the effect of the applied jet. Figures 5 (a, b and c) and 6 (a, b and c) report the variation of the scour profiles with time advance in the case of wall jet with 1:1 and 3:1 downstream face slope, respectively. They show an increase of the scour depth value with



a) Discharge 2.19·10⁻³ m³/s; Yₜ=3.02 cm

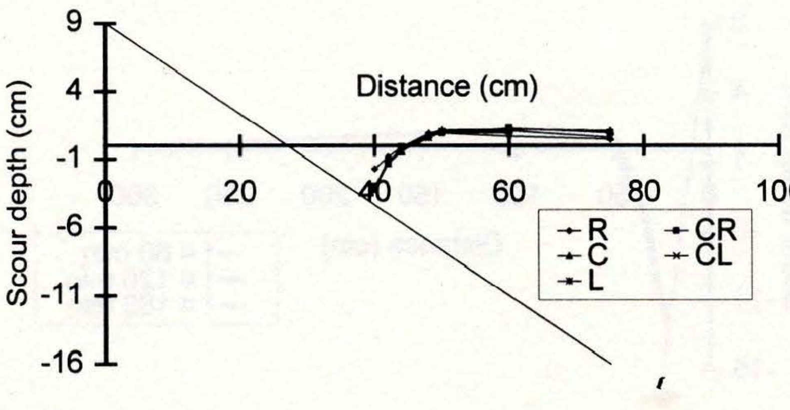


b) Discharge 2.73·10⁻³ m³/s; Yₜ=7.95 cm

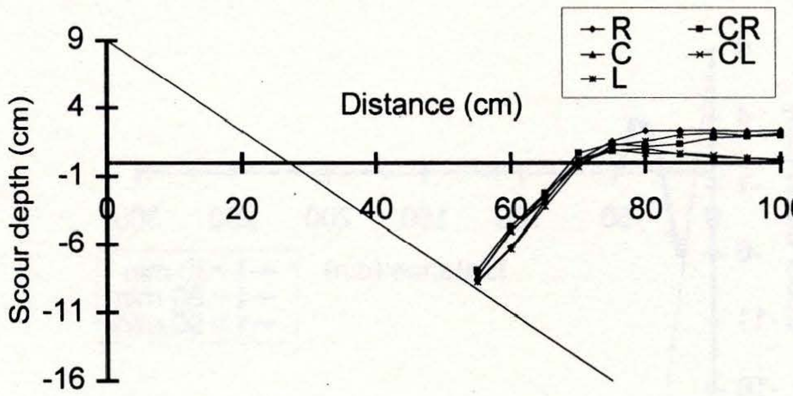


c) Discharge 4.04·10⁻³ m³/s; Yₜ=7.18 cm

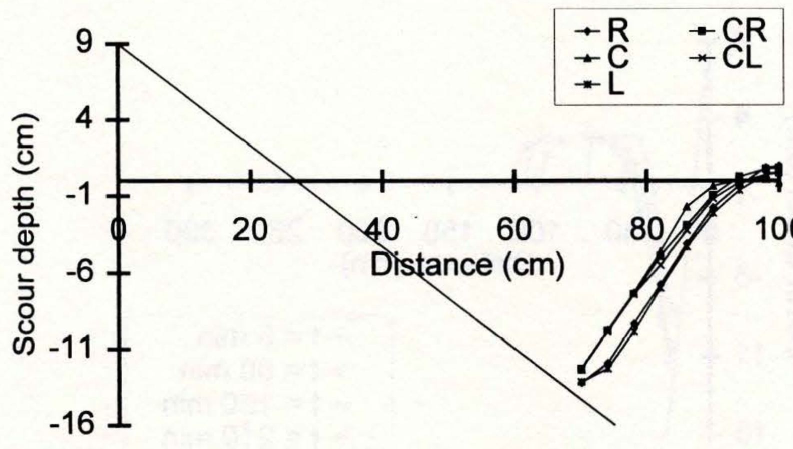
Fig. 3. Equilibrium scour profiles (λ=45°)



a) Discharge $1.36 \cdot 10^{-3}$ m^3/s; $Y_t=2.54$ cm

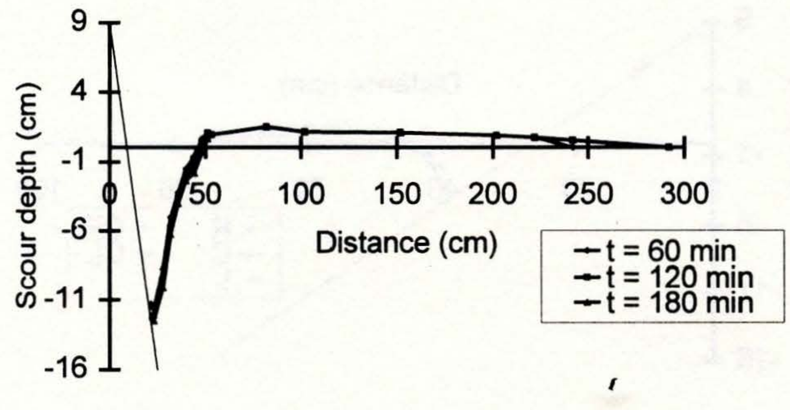


b) Discharge $2.62 \cdot 10^{-3}$ m^3/s; $Y_t=3.01$ cm

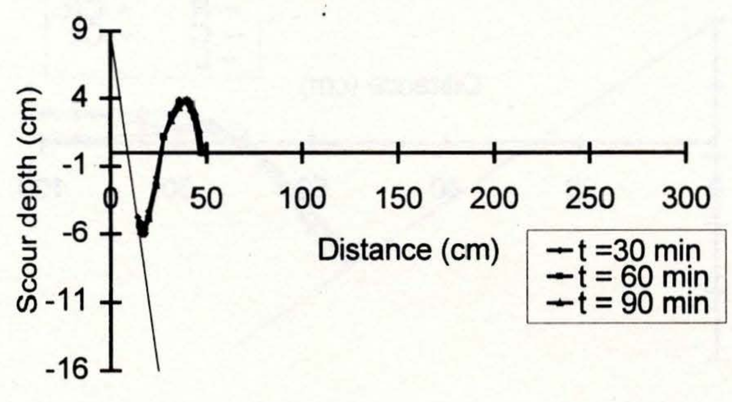


c) Discharge $3.88 \cdot 10^{-3}$ m^3/s; $Y_t=3.53$ cm

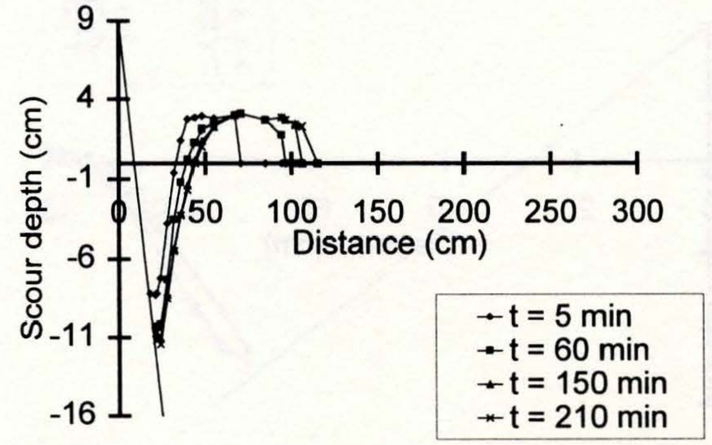
Fig. 4. Equilibrium scour profiles ($\lambda=18°$)



a) Discharge $2.19 \cdot 10^{-3}$ m^3/s; Y_t=3.02 cm

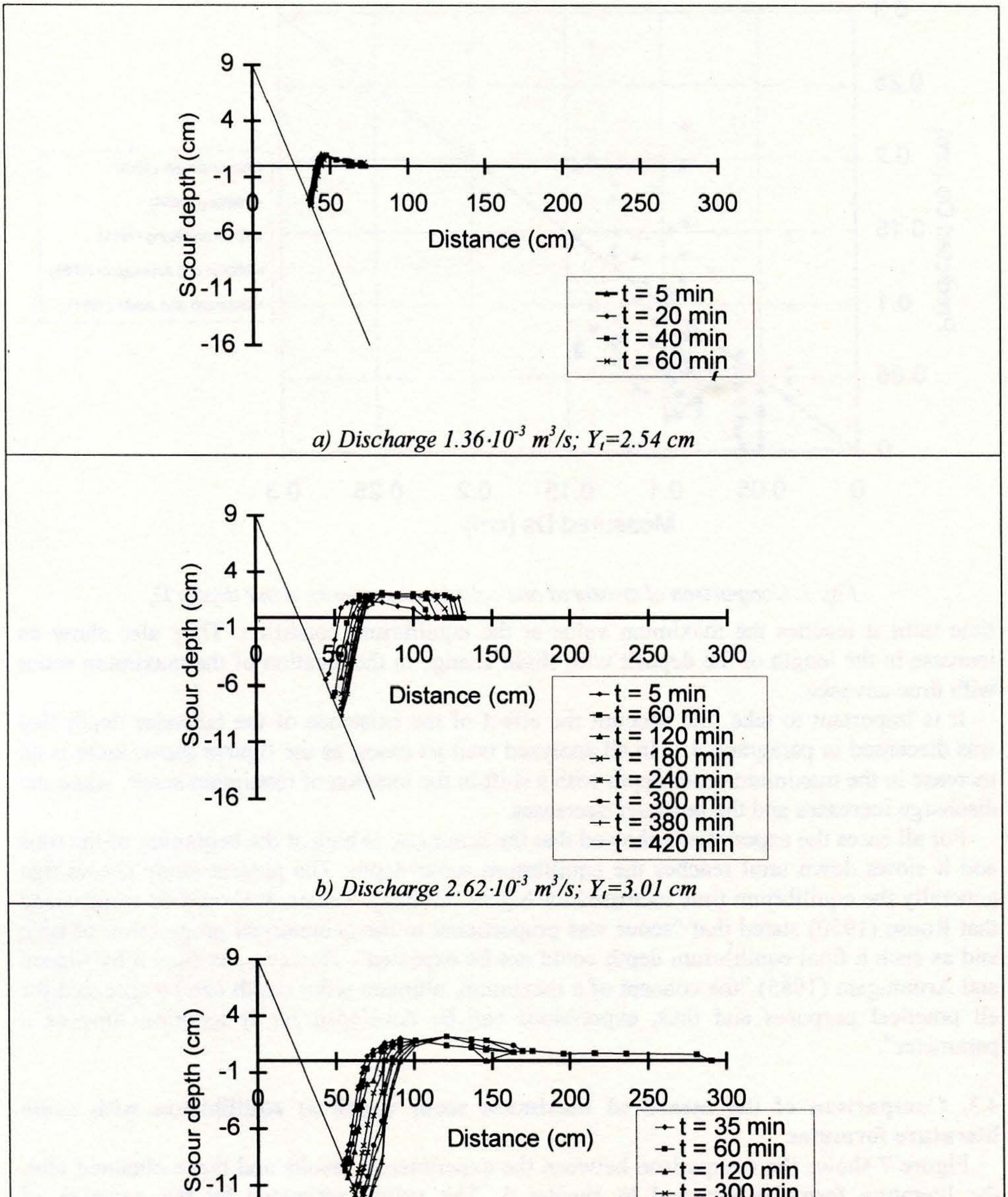


b) Discharge $2.73 \cdot 10^{-3}$ m^3/s; Y_t=7.95 cm



c) Discharge $4.04 \cdot 10^{-3}$ m^3/s; Y_t=7.18 cm

Fig. 5. Variation of scour profiles with time (λ=45°)



a) Discharge $1.36 \cdot 10^{-3}$ m^3/s; Y_t=2.54 cm

b) Discharge $2.62 \cdot 10^{-3}$ m^3/s; Y_t=3.01 cm

c) Discharge $3.88 \cdot 10^{-3}$ m^3/s; Y_t=3.53 cm

Fig. 6. Variation of scour profiles with time (λ=18°)

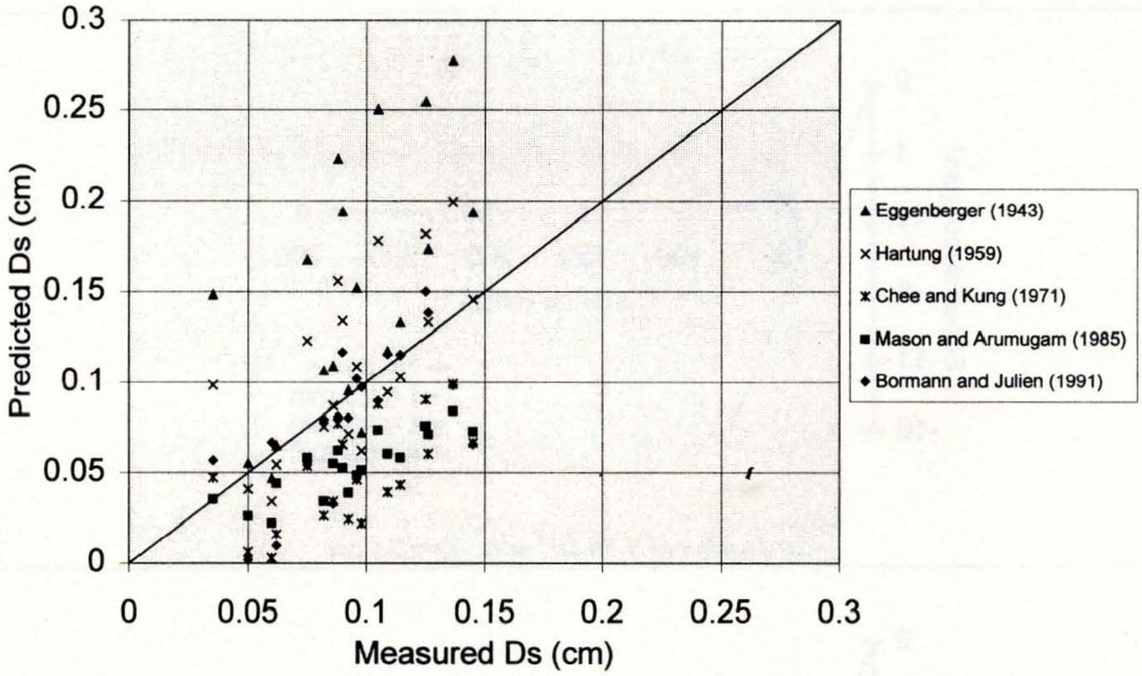


Fig. 7. Comparison of measured and calculated maximum scour depths D_s

time until it reaches the maximum value at the equilibrium condition. They also show an increase in the length of the deposit with slight change in the location of the maximum scour with time advance.

It is important to take into account the effect of the existence of the tailwater depth that was discussed in paragraph 4.1. In all analyzed wall jet cases, as the figures show, there is an increase in the maximum scour depth with a shift in the location of maximum scour, when the discharge increases and the tailwater decreases.

For all cases the experiments showed that the scour rate is high at the beginning of the runs and it slows down until reaches the equilibrium scour depth. The present study shows that generally the equilibrium time increases for higher discharge values. It should be emphasized that Rouse (1950) stated that "scour was proportional to the geometrical progression of time and as such a final equilibrium depth could not be expected". However, as shown by Mason and Arumugam (1985) "the concept of a maximum, ultimate scour depth can be accepted for all practical purposes and thus, expressions can be developed for it ignoring time as a parameter".

4.3. Comparison of the measured maximum scour depth at equilibrium with some literature formulae

Figure 7 shows the comparison between the experimental results and those obtained with the literature formulae reported in chapter 3. The values estimated by the equation of Eggenberger (1943) were higher than the experimental results. The equation of Hartung (1959) provided a fair agreement with the experimental results, while those estimated by the equations of Chee and Kung (1971) and Mason and Arumugam (1985) were lower than the experimental results.

The equation of Bormann and Julien (1991) was applied for the current study by using the $\alpha=\beta'$ (see, for example, Rajaratnam, 1981), $\Phi=30°$ and $B=2.9$. The values of β' were calculated following the regression equation proposed by Bormann and Julien (1991). The proposed value of B was founded to better estimate the experimental results with that formula. It is important to highlight that in the formula of Bormann and Julien (1991) the coefficient K depends on the analyzed configuration and the sediment properties as shown by eq. (2). Figure 7 shows that the experimental results agree rather well with the estimated ones, taking into account, on the one hand, the sensitivity of the scour equation to the K coefficient, and on the other hand, the complexity of the physical phenomena.

5. DIMENSIONAL ANALYSIS OF THE LOCAL SCOUR DOWNSTREAM OF GRADE-CONTROL STRUCTURES

Generally the scour formulae proposed in the previous chapter are not dimensionally homogeneous. This should be a warning that the formulae change if the units change and they represent some special cases that each author studied.

By applying the dimensional analysis to the current case study, the maximum scour depth at equilibrium state can be expressed as a function of different parameters as follows:

$$D_s/Y_0 = f\left(Re, Wb, Fr_d, Fr_s, K_u, \left(Y_s + D_p\right)/Y_s, Y_s/Y_t, \beta, V_e/U_0\right) \tag{3}$$

where Y_0 is the jet thickness entering the tailwater, Re is the Reynolds number, Wb is the Weber number, $Fr_d=U_b/(gd_s(\gamma_s-\gamma)/\gamma)^{0.5}$ is the densimetric Froude number where U_b is the diffused jet velocity at the original bed location, d_s is the sediment size and β is the angle of the jet entering tailwater. The scour depends also on a number of factors such as submergence and the level of the bed relative to the apron, which can be taken into account with the dimensionless parameters $(Y_s+D_p)/Y_s$ and Y_s/Y_t, where D_p is the drop height of the structure, the degree of turbulence dissipation of the jet, that can be expressed by the turbulence index K_u of the jet entering tailwater, air entrainment that can be expressed by the ratio V_e/U_0, where V_e is the velocity at which air entrainment commences. Using the analysis of the jet trajectory, shown in Fig. 2, the jet velocity entering tailwater U_0 can be derived as follows:

$$U_0 = \left(U_s^2 + 2g\Delta H\right)^{0.5} \tag{4}$$

where ΔH is the fall from sill level to tailwater level, assumed equal to $D_p+Y_s/2-Y_t$.

It must be emphasized that the approximate solution of eq. 4 is obtained by neglecting the non-uniform velocity distribution in the approach flow region and the effects of air bubble entrainment (Chanson, 1996). Furthermore, the tailwater becomes a mixture of water, entrained air and scoured bed material induced by the jet. In the present study only wall jet cases were analyzed for which β is assumed equal to λ. Furthermore, it is assumed that the jet were accelerated under the influence of gravity.

Referring to plane turbulent jet issuing into the same fluid, Albertson et al. (1950) proposed the following equation

$$U_m/U_0 = 3.24\left(2s/Y_0\right)^{-0.5} \tag{5}$$

for the jet diffusion in the established flow region (see Rajaratnam, 1976), where s is the longitudinal distance of the jet starting from the cross section in which it enters the tailwater,

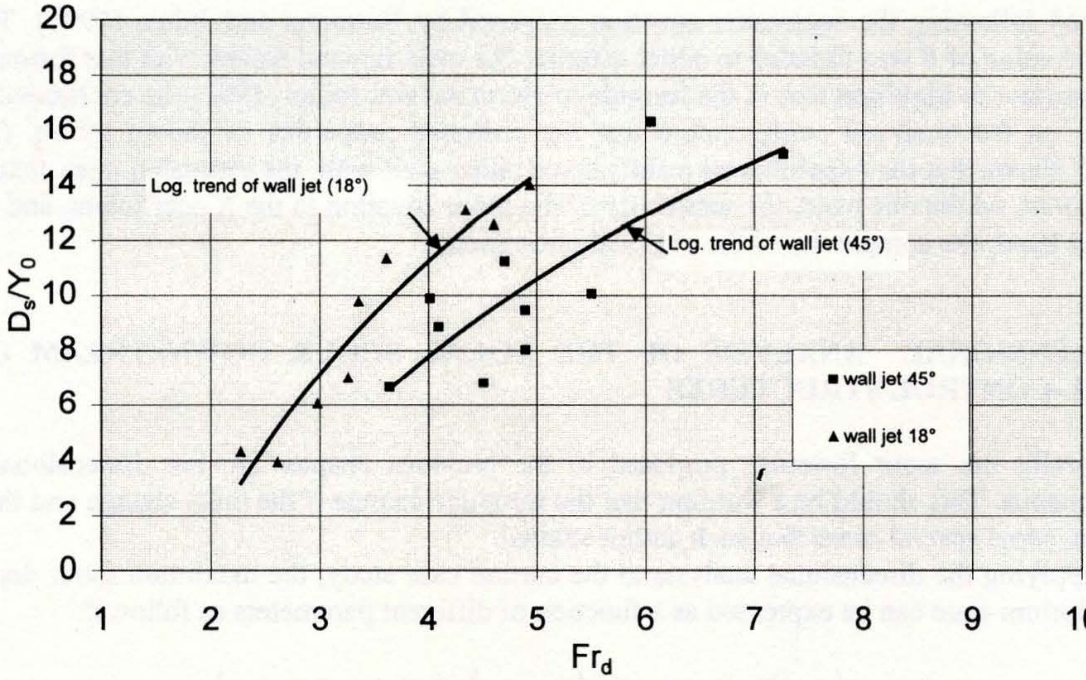


Fig. 8. Ratio of the measured scour depths and Y_0 versus Fr_d

U_m is the diffused jet velocity at distance s. The diffused jet velocity near the original bed U_b can be calculated by using eq. (5) with $s=Y_t/\sin(\beta)$.

Referring to eq. (3), the maximum scour depth D_s can be expressed as a function of the densimetric Froude number Fr_d and the angle of the jet entering tailwater. Figure 8 shows the experimental maximum equilibrium scour depths D_s nondimensionalized by Y_0 versus Fr_d. The logarithmic trend curves of the two wall jet cases are also reported. The experimental data of the present study are characterized by the angle of the jet entering tailwater equal to 18°, 45° and, moreover, $(Y_s+D_p)/Y_s$ and Y_s/Y_t are on the average equal to 7.20 and 0.29, respectively. In this analysis we neglect the effects of K_u, that is really essential when turbulence generators are used (Chiaia et al, 1997), Weber and Reynolds numbers and air entrainment.

Figure 8 clearly shows that the analyzed phenomenon depends on the face angle of the structure, that is the impingement angle of the jet. The comparison of the measured and calculated maximum scour depths has been carried out, by defining a relative error ε as follows

$$\varepsilon = \left|(D_s)_{measured} - (D_s)_{calculated}\right| / (D_s)_{measured} \,. \tag{6}$$

The experimental results are characterized by a mean relative error equal to 15% by using the logarithmic equation of D_s/Y_0.

6. CONCLUSIONS

The problem of the scour downstream of the hydraulic control structure is of great importance, because it can affect the stability of the structure.

The current study deals with the scour process downstream of a grade control structure. The experiments were carried out at the hydraulic laboratory of the Mediterranean Agronomic Institute of Bari (Italy). The paper presents the scour process for the conditions of wall jet cases with different tailwater depths.

Considering the analysis of the experimental results the following conclusions can be formulated:

- For all experiments with different configurations of both jet diffusion and jet trajectory, the maximum scour depth was found to increase with time advance, performing a higher scouring rate at the beginning of the experiment and then slowing down until it reaches an equilibrium state.
- The final scour profiles along five longitudinal sections located parallel to the flow direction in the flume are quite similar, which means that the wall effect can be neglected in the runs of the present study.
- The scour profiles show an increase in the maximum scour depth with time advance, and an increase in both the height and the length of the deposit part.
- The effect of the tailwater depth for the jet diffusion was experimentally showed. For the same structure different jet diffusion configurations were reached by changing the tailwater depth in the channel. It was found that the higher the tailwater depth, the lower the value of the maximum scour depth.
- The experimental data of the maximum scour depth were compared with the results of some main literature formulae. Particularly, Bormann and Julien (1991) proposed to calculate the parameter K in scour equation taking into account the inlet geometry and the sediment properties. Nevertheless, using the dimensional analysis of the phenomenon, a different approach is proposed in the present study providing a fair agreement with the experimental results.
- The experimental results show that scour downstream of grade-control structure depends on the face angle of the structure, that is the impingement angle of the jet.

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