AN ANALYSIS OF SPH SMOOTHING FUNCTION MODELLING A REGULAR BREAKING WAVE

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ABSTRACT

The Smooothed Particle Hydrodynamics is a Lagrangian mesh-free particle model introduced by Gingold and Monaghan (1977). In this paper, the SPH Kernel is analyzed in order to find measures of merit for two-dimensional SPH.
The smoothing function plays a very important role in the SPH approximations, as it determines the accuracy of the function representation and the efficiency of the computation. The generalized approach in constructing the smoothing functions for the SPH method uses an integral form of function representation with the support of the Taylor series expansion.
In addition to theory, comparisons with physical model runs are analyzed, demonstrating the important role of the smoothing function in terms of computational accuracy. The SPH model is applied to the modelling of water waves generated in the wave flume of the Water Engineering and Chemistry Department laboratory of Bari Technical University (Italy). It is shown that the final version is able to model the propagation of regular and breaking waves.

1 INTRODUCTION

The numerical technique (SPH) is a gridless, pure Lagrangian method for solving the equations of fluid dynamics.

In this paper we analyze the key element in the Smoothed Particle Hydrodynamics (SPH) method, the SPH Kernel, in order to develop a measure of merit for evaluating Kernels in two dimensional SPH.

The main features of the SPH method, which is based on integral interpolations, were described in detail by *Monaghan* (1982), *Benz* (1990), *Monaghan* (1992) and *Liu* (2003).

The alternative view is that the fluid domain is represented by nodal points that are scattered in space with no definable grid structure and move with the fluid. Each of these nodal points carry scalar information, density, pressure, velocity components and so on. To find the value of a particular quantity f at an arbitrary point, x, we apply an interpolation:

$$f(x) = \sum_j f_j W(x - x_j) V_j \tag{1}$$

Here f_j is the value of f associated with particle j, located at x_j, $W(x-x_j)$ represents a weighting of the contribution of particle j to the value of $f(x)$ at position x, and V_j is the volume of particle j, defined as the mass, m_j, divided by the density of the particle ρ_j. The weighting function, $W(x-x_j)$, is called the Kernel and varies with the distance from x.

The Kernel is assumed to have compact support, so the sum is only taken from neighboring particles. Some background in SPH is assumed in this paper; readers are referred to an overview by *Monaghan* (1992), while for recent applications of SPH see *Dalrymple & Rogers* (2006). To some extent it should not matter which Kernel is used in SPH as long as basic requirements are met. This is especially true in the limits where h (the Kernel smoothing length) and Δx (the interparticle spacing) become small.

However, when these are not small, as is common in practice, the choice of Kernel can drastically change the computational results. Hence, the choice of Kernel, h and Δx is a key decision before performing any calculation using SPH. This paper provides an objective means of separating better from poorer Kernel performance in terms of $\Delta x/h$ value.

In performing the analysis we consider the cubic spline Kernel and its first derivative.

Monaghan & Lattanzio (1985) devised the following function based on the cubic spline function known as the B-spline function:

$$W(R,h) = \alpha_d \times \begin{cases} \frac{2}{3} - R^2 + \frac{1}{2}R^3 & 0 \le R < 1 \\ \frac{1}{6}(2-R)^3 & 1 \le R < 2 \\ 0 & R \ge 2 \end{cases} \tag{2}$$

where $R = r_{ij}/h$, $r_{ij}=|x-x_j|$ and $\alpha_d = 1/h$ for 1D, $\alpha_d = 15/\pi h^2$ for 2D, and $\alpha_d = 3/2\pi h^2$ for 3D.

The cubic spline function has so far been the most widely used smoothing function in emerging SPH literature, since it resembles a Gaussian function while having a narrower compact support.

The SPH model is applied to the modelling of water waves generated in the wave flume of the Water Engineering and Chemistry Department laboratory of Bari Technical University (Italy).

2 NUMERICAL TESTS AND EXPERIMENTAL SET-UP

The efficiency and accuracy of the constructed smoothing functions have also been shown in various literatures for all existing smoothing functions. Readers are referred to work by *Fulk & Quinn* (1995).

Presented here are two numerical examples using the cubic spline function of equation (2).

The implemented numerical code was first tested using physical experiments on wave motion fields by *De Serio & Mossa* (2006).

The experiments were performed in a wave channel 45 m long and 1 m wide. The iron frames supporting its crystal walls are numbered from the shoreline up to the wavemaker (section 100), thus locating measurement sections which have a center to center distance equal to 0.44 m. From the wave paddle to section 73 the flume has a flat bottom, while from section 73 up to the shoreline it has a 1/20 sloped wooden bottom. A sketch of the wave flume is shown in Fig. 1.

Further details about the experimental tests carried out can be found in *De Serio & Mossa* (2006).

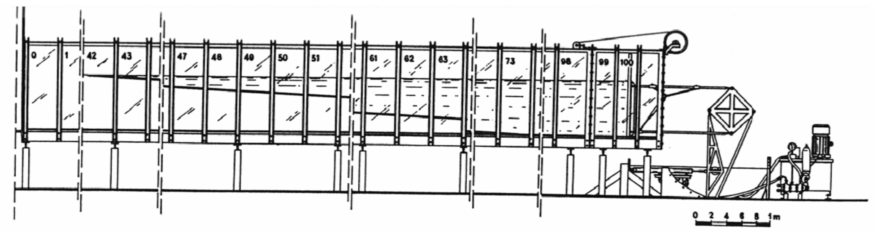

Figure 1. Sketch of the wave flume.

The water depth, the wave height and the period were equal to 0.70 m, 0.11 m and 2s, respectively, in section 0.5 m offshore section 76 (Fig. 2).

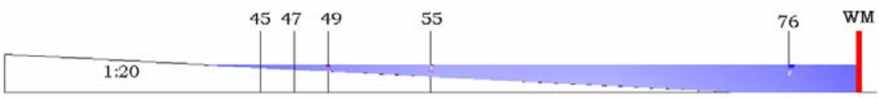

Figure 2. Measurement sections.

The simulations in the present paper used an artificial viscosity with an empirical coefficient α equal to 0.055 (*Monaghan*, 1992). Each wall in the computational wave tank was built with two parallel layers of fixed boundary particles set out in a staggered manner described by *Dalrymple & Knio* (2000). In this approach the boundary particles share some of the properties of the fluid particles, but their velocities are zero and their positions remain unchanged. The choice of the $\Delta x/h$ term depends on the physical process of the problem and the desired computational accuracy and efficiency. However, if an interval of $\Delta x/h$ value is studied, the quality of Kernel as particle movement can be deduced. As shown in *Fulk & Quinn* (1995), for almost every Kernel, the results start to become relatively poor when $\Delta x=h$. However, we verified this result for SPH calculations. The particle spacing is taken as $\Delta x=\Delta z=0.022$ m and thus approximately 30,000 particles are used. For a comparison between computational accuracies, we used a smoothing length of $h=0.0305$ m and $h=0.0212$ m and, thus, a value of $\Delta x/h=0.7213$ and $\Delta x/h=1.0377$ was used (Table 1).

Test	Time Simulation [s]	Particle number	$\Delta x/h$
1	20s	30,000	0.7213
2	20s	30,000	1.0377

Table 1. Characteristics of SPH simulations.

The study made particular reference to the velocity and free surface elevation distributions with the aim of analysing the performance of Kernel in terms of stability in the fluid. The final model proved capable of reproducing the experimental propagation of regular and breaking waves.

3 RESULTS

The experimental and numerical wave profiles at the location of measurement points are shown for both cases. For the sake of brevity only the results of sections 55-49 (see Fig. 2) will be shown. For a defined section, we can study the distribution along the channel of the wave elevation and the horizontal and vertical velocity components for both tests.

Figures 3a÷3c and 4a÷4c show the agreement of numerical data obtained by means of the two SPH models with experimental data. In Figs. 3a÷3c and 4a÷4c it can be seen that the numerical elevations and the numerical velocities obtained by means of the second test of Table1, are not in perfect agreement with the experimental measurements for the strong effect of the $\Delta x/h$ term. In fact, when this value is close to 1 (Test 2), the computational results can drastically change for the worse.

With a smaller value of the $\Delta x/h$ term (Test 1), numerical elevations and the numerical velocities are shown to be in better agreement with the experimental measurements (Figs. 3a÷3c and 4a÷4c).

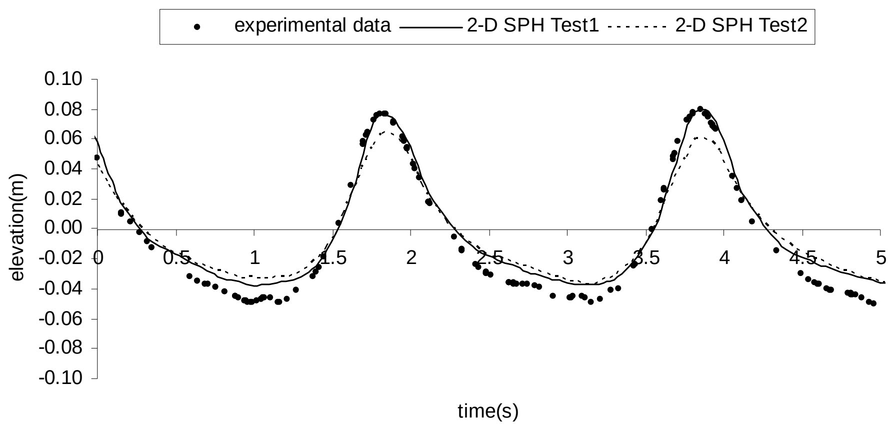

Figure 3a. Comparison of experimental and numerical wave surface elevation (section 55).

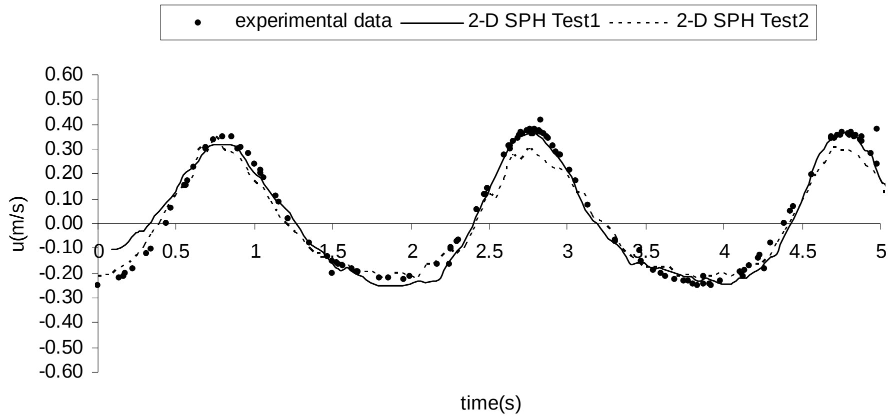

Figure 3b. Comparison of experimental and numerical horizontal velocity components (section 55, 0.1 m from the bottom).

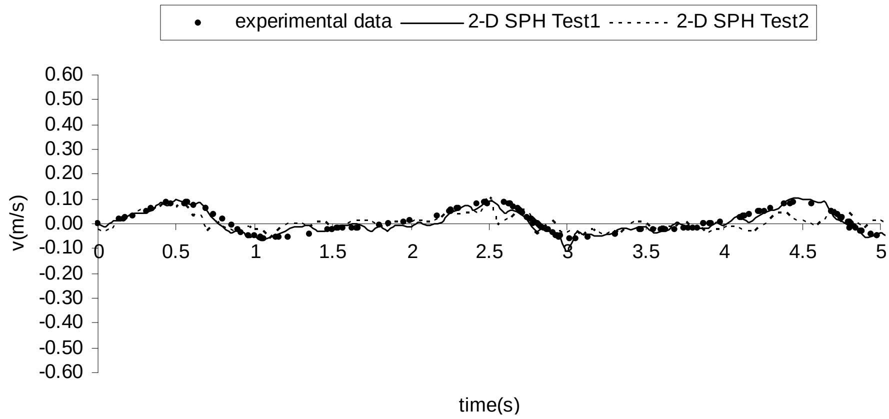

Figure 3c. Comparison of experimental and numerical vertical velocity components (section 55, 0.1 m from the bottom).

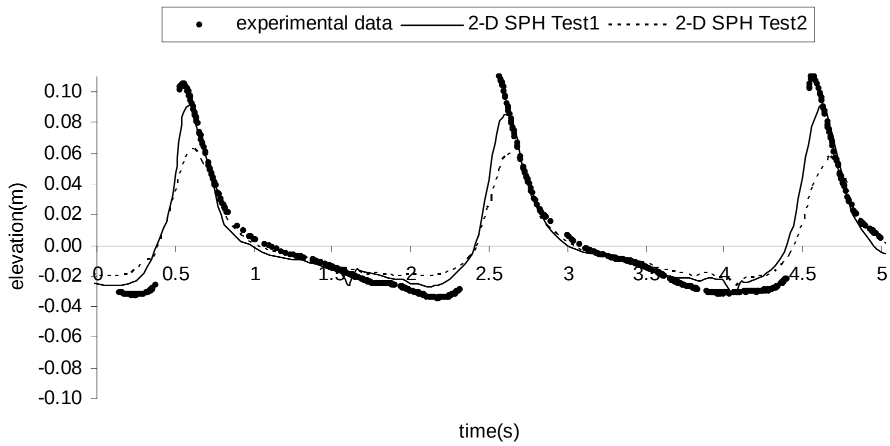

Figure 4a. Comparison of experimental and numerical wave surface elevation (section 49).

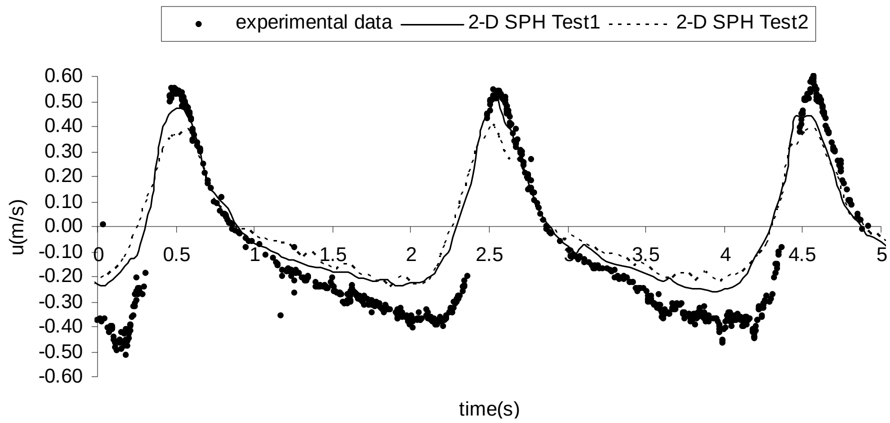

Figure 4b. Comparison of experimental and numerical horizontal velocity components (section 49, 0.1 m from the bottom).

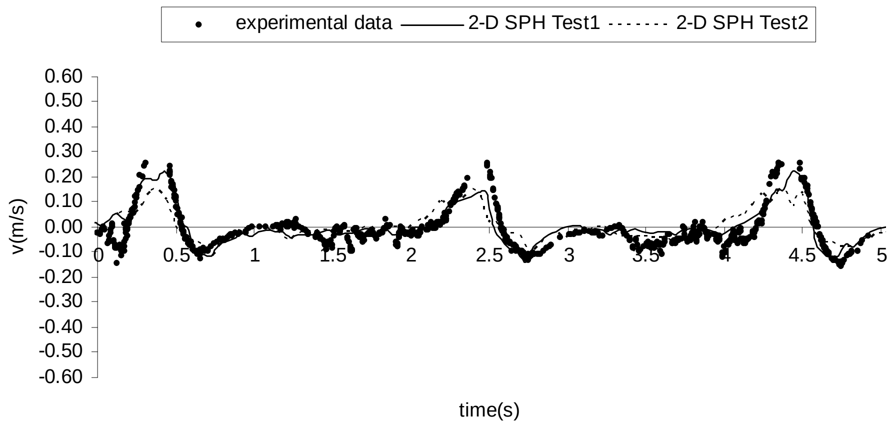

Figure 4c. Comparison of experimental and numerical vertical velocity components (section 49, 0.1 m from the bottom).

Although Figs. 3a ÷ 3c and 4a ÷ 4c provide good qualitative results, it is desirable to obtain quantitative results as well. Overall statistical parameters can provide a more detailed picture of the breaking model performance. Figure 5 shows standard deviations of measured and computed surface elevations of sections 76, 55, 49, 47 and 45 (Fig. 2).

Skewness (*Kennedy et al.,* 2000), a measure of crest-trough shape, is computed and shown in Fig. 6. Test 1 predicts this parameter very well; in fact the trend of wave skewness increases as the wave shoals and breaks, and decreases near the shoreline (section 49). Instead, in the case of Test 2 of Table 1, when the value of $\Delta x/h$ term is close to 1, the computational results change for the worse and the trend of wave skewness is not well predicted, in particular in sections 47 and 45, where the wave surface profiles are characterized by a rapid change in shape.

This result shows how a good efficiency of the Smoothed Particle Hydrodynamics Kernel is not obtained for all values of $\Delta x/h$.

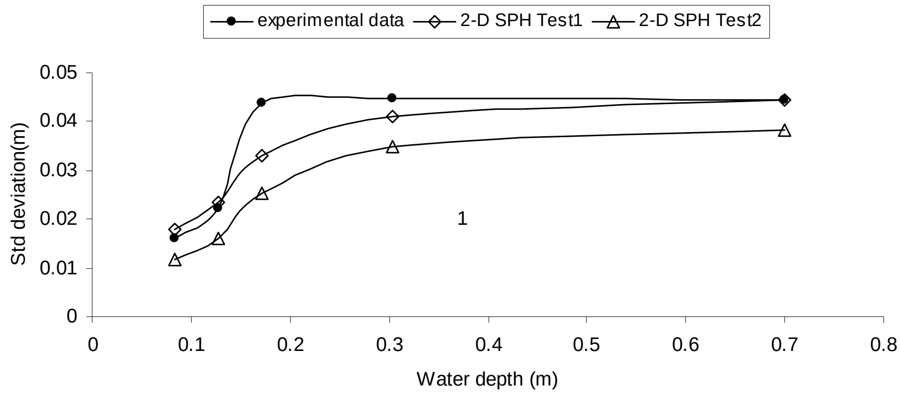

Figure 5. Comparison of experimental and numerical standard deviation of surface wave elevations.

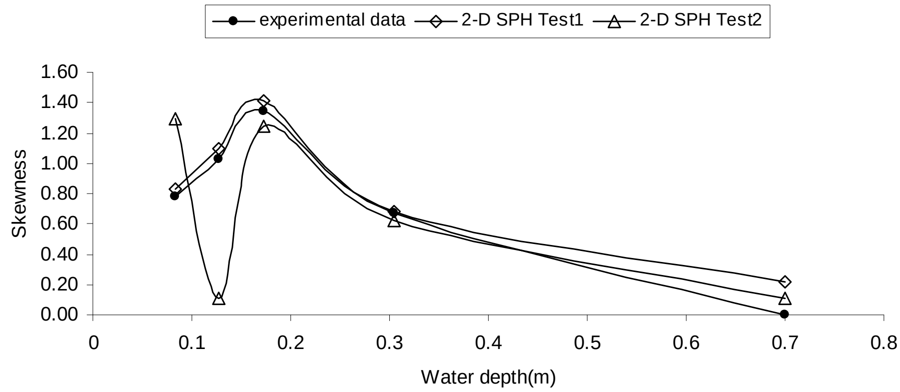

Figure 6. Comparison of experimental and numerical skewness of surface wave elevations.

4 Conclusions

This paper presents the modelling of the propagation of regular and breaking waves using the SPH approach. Comparisons with physical model runs are analyzed, with the aim of showing how the efficiency of the SPH Kernel depends on the choice of the $\Delta x/h$ term. In the runs of the present paper, we observed that in cases with a higher value of the $\Delta x/h$ term, the numerical wave surface elevations and velocities were not in good agreement with the experimental ones. The results with a $\Delta x/h$ value lower than 1 show a better reproduction of the experimental values.

These results highlight the fact that, for a certain Smoothed Particle Hydrodynamics Kernel, it is important to define and use a correct value of the $\Delta x/h$ term in the model. This is particularly useful in selecting the initial particle separation for a given Kernel.

Therefore, generally speaking, an appropriate value of $\Delta x/h$ term should be used for a settled Kernel in order to obtain good results.

REFERENCES

Benz, W. (1990). Smoothed particle hydrodynamics. A review The numerical modelling of nolinear stellar pulsations: Problems and prospects, J. R. Butchler, ed., Kluwer Academic, Dordrecht, The Netherlands, 1990, pp. 264-288.

Dalrymple, R. A., and Knio, O., SPH modelling of water waves. Proc., *Coastal Dynamics,* 2000, pp. 779-787.

Dalrymple, R. A., and Rogers, B.D., Numerical modelling of waves with the SPH Method. Proc., *Coastal Engineering*, 2006, pp. 53, 131-147.

De Serio, F., and Mossa, M., Experimental study on the hydrodynamics of regular breaking waves. *Coastal Engineering*, 2006, 53 pp. 53, 99-113.

Fulk, A., and Quinn, D. W., An Analysis of 1-D smoothed Particle Hydrodynamics Kernels, *Journal of Computational Physics,* 1995, 126, pp.165 - 180

Gingold, R. A., and Monaghan, J. J., Smoothed particle hydrodynamics: Theory and application to nonspherical stars. *Mon. Not. R. Astron. Soc.*, 1977, 181, pp. 375–389.

Kennedy, A. B., Chen, Q., Kirby, J. T., and Dalrymple, R. A., Boussinesq Modelling of Wave Transformation, Breaking. And Runup. *Journal of Waterway,Port, Coastal, and Ocean Engineering,* 2000, 126, pp.39 - 47

Liu, G. R., Mesh free methods: Moving beyond the finite element method, *Chemical Rubber, Boca Raton, Fla.,* 2003.

Monaghan, J.J., Why particle methods work, SIAM (Soc. Ind. Appl. Math) *J. Sci. Stat. Comput.*, 1982, 3, pp 422-433.

Monaghan, J.J., and Lattanzio J. C., A refined particle method for astrophysical problems. *Astronomy and Astrophysics*, 1985, 149, pp.135-143.

Monaghan, J.J. Smoothed particle hydrodynamics. *Annual Review of Astronomy and Astrophysics* 1992, 30, pp. 543–574.