

Experimental Study on the Estimation Methods of Wave Orbital Velocity

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ABSTRACT

The kinematics of a regular wave field has been investigated in order to determine the best mathematical representation. The paper describes the comparison between many theoretical models reported in literature and the experimental results obtained in a bidimensional channel at the laboratory of the Department of Water Engineering of the Polytechnic of Bari. First of all the classic wave theories have been discussed in order to evaluate the horizontal and vertical velocities. Then, the theoretical models (linear and non-linear), which link wave elevations and velocity components, have been tested by comparing the numerical results with the laboratory LDA measurements carried out in a channel with a sloped bottom. In the present paper the models proposed by Chakrabarti and by Koyama and Iwata have been applied, giving a relationship between time series of velocity components and wave elevations. Furthermore, a linear transfer function between the elevation amplitude spectra and the orbital velocity component ones has been investigated. The waves generated in the channel are regular (i.e. waves with a permanent form) non-linear waves which can be approximated with 2nd order Stokes in the deeper water sections of the channel, following the classic criteria.

KEY WORDS: Regular waves; Non-linear waves; Engineering approximations; Kinematics; Wave orbital velocity.

INTRODUCTION

A reliable description of the wave kinematics is very important to know the dynamic action of waves on structures and to predict coastal sediment transport processes. In order to obtain a correct understanding of the phenomena, many authors have recently proposed mathematical models and approximate methods able to describe the wave motion field both in deep and shallow water.

The problem has been well known since the last century, when the first classic theories were developed. Although these theories are

still actual, they imply some hypotheses which are often not realistic and, thus, they need to be reviewed in the light of new numerical and experimental studies.

Relevant mathematical models can simulate the regular waves quite well within a range of validity in which the aforementioned hypotheses are quite acceptable. Regular waves can be reproduced in laboratory and are characterised by a constant form in time domain. Lately new methods for estimating wave orbital velocity have been studied in the light of experimental observations. These studies are preliminary for random waves which indeed can be studied by extending results on regular waves.

This paper presents a study of the transfer functions linking the local time history of elevations with the local velocity field. This procedure is useful because it is easier to measure in situ wave elevations (or to forecast them with statistical methods) than the velocity components. In recent years some methods which link wave elevations with velocity components have been studied, although the theoretical results are not yet supported enough by experimental results.

The merit of some of these methods is the simplification of the mathematical procedures. In fact, the increase of difficulties in a mathematical solution, to better the simulation of physical processes, does not always mean more reliable results. Moreover, some authors (Dean, 1970; Graw, 1994) agree that it is not always possible to evaluate all the characteristics of a wave field by using only one method with the same level of accuracy. On the contrary, it can be possible, for the same wave field, that a theory approximates a quantity better, and another one can be successfully used to interpret other quantities.

The aim of this paper is to investigate the wave velocity results obtained with some mathematical models and compare them with experimental measurements to determine when to use one model rather than another one. The models were used to interpret the same waves moving in a channel with a sloped bottom, where shoaling effects are evident.

The comparison of calculated velocities with the measured ones can be carried out both in time and frequency domains. In the present paper theories and measurements were compared in frequency domain, by defining a relative error.

THEORETICAL SOLUTIONS

Firstly, classic theories have been applied to obtain reference results for modern methods. To this purpose 2nd and 3rd order Stokes have been applied according to the typical classification criteria. Indeed almost all the investigated waves fall in the range of validity of 2nd and 3rd order Stokes theory.

Stokes waves have been studied by many researchers who gave different mathematical solutions. *Fenton (1985)* gave a decisive contribution to the understanding of fifth-order Stokes theory for steady waves. He stated that the waves travel at Stokes' first definition of wave speed. In Fenton's solution, some of the disadvantages of previous theories have been shown by assuming the expansion parameter ak in which a , having no physical significance other than that of being a length scale, is supposed to be equal to $H/2$. In such a way the expressions for the coefficients of higher order terms are functions of the dimensionless depth kh , so that the only unknown is the wave number k . The suggested procedure ensures the convergence of the series expansions for waves approaching the maximum steepness as observed by *Rebaudengo Landò and Scarsi (1995)*, who applied Fenton's assumptions to develop a non-linear model for directional random waves. As Fenton stated, the classic *Skjelbreira and Hendrikson (1960)* solution for higher order Stokes waves contains errors of the fifth order; moreover the theory suggested by Fenton has been shown to be quite accurate for waves shorter than 10 times the water depth.

The waves investigated in this study, as already said, can be generally approximated with the 3rd order Stokes theory. In this study a third order solution as reported in *Sawaragy (1995)* has been used while the equation in *CERC (1984)* has been employed to calculate the wave length.

The previous theories allow the prediction of both elevations and velocities, once depth, wave height and length (or period) are known; for practical purposes, it can be useful to evaluate the velocity field starting from elevation time history with the following equations:

$$\begin{aligned} u(t) &= H_u(h, T, Z) \eta(t) \\ w(t) &= H_w(h, T, Z) \eta(t + \Delta t) \end{aligned} \quad (1)$$

in which H_u and H_w are the transfer functions and the terms u and w are the horizontal and vertical orbital velocity components respectively, η is the wave elevation, Z is the distance from the bottom, h is the local depth, H and T are the wave height and period, k is the wave number and Δt represents the delay between the wave elevation values and vertical velocity ones, assumed to be equal to $T/4$ according to classic theories. It should be mentioned that the theoretical phase shifting among velocity components and elevations, assumed to be zero for the horizontal component and $\pi/2$ for the vertical one, should be reviewed in the light of the recent experiences (*Damiani and Mossa, 1996b*) showing different values of the aforementioned shifting. Sometimes it can be useful to replace the second equation (1) with a relationship between u and w by introducing the function H_{u-w} which links the two velocity components. Thus the problem is the research of the better transfer functions linking the wave elevations with the velocity field.

The transfer functions can assume a linear form (*Koyama and Iwata, 1986*); in this way, if the elevations follow the small amplitude wave theory, the equations (1) become formally identical to Airy formulation. For non-linear waves *Koyama and Iwata* suggested a modified equation for the transfer function of the first equation (1), to evaluate horizontal velocity under the crests.

Consequently, also the vertical velocity is modified by applying the function H_{u-w} .

The later two methods give a description of velocity field once the time history of wave elevations is known. It can be useful to decompose the wave surface profile into Fourier components.

Woltering and Daemrich (1995) applied the linear theory to each frequency component of a regular wave elevation spectrum obtaining the velocity component spectra. Basically they treated the spectrum like a superposition of small amplitude waves still resulting in a small amplitude wave. In this procedure, for each frequency component, a transfer function assuming the same equation proposed by *Koyama and Iwata* has been applied. The linear method of *Koyama and Iwata* gives the same results of the procedure of *Woltering and Daemrich* only if the considered waves are monochromatic, that is when the spectrum is characterized by only one peak.

Woltering and Daemrich (1994) suggested also a Lagrangian approach to take into account the non-linearities in Stokes waves. They found that for wave groups the linear method overestimated the experimental velocities under the crests, while the non linear method gives more reasonable results.

Although Stokes' theory is not valid above the mean water level, it has often been used to find the velocity components between the trough and the crest. Measurements of wave kinematics indicate that the horizontal velocity is smaller at the crest and higher in the trough than predicted by Stokes higher order theories (*Gudmestad and Connor, 1986*). Many engineering approximate methods have been developed in order to go over these limits.

Gudmestad and Connor (1986) report some of these methods for non linear deep water waves, consisting in empirically modified small amplitude wave theory equations (*Wheeler and Chakrabarti* methods) or in assuming a constant velocity potential above the still water level (*Mo and Moan* method). The first two methods predict higher trough velocities and lower crest velocities than the linear theory and the higher order Stokes theories. *Gudmestad and Connor* proposed also an approach to obtain second order correction terms to the engineering approximations, by following an expansion procedure. Particularly, in the present paper *Chakrabarti (1971)* method and its second order approximation (*Gudmestad and Connor, 1986*) have been applied to interpret the experimental velocity measurements in the whole wave field. The experiments of *Gudmestad and Connor* confirm the validity of the second order expansion of *Chakrabarti* method.

All the previous theories have been extended to random waves by *Vis (1980)*, *Guza and Thornton (1980)*, *Rebaudengo Landò and Scarsi (1995)* and other Authors. The former analysed the linkage between wave elevation amplitude spectra and velocity component ones. They found a good agreement with experimental results by using a linear transfer function not only for narrow band spectra, but also for spectra with a remarkable spreading in frequency or for clearly non linear waves.

Rebaudengo Landò and Scarsi (1995) developed a third order model able to describe both time history of the surface elevations and wave kinematics of unidirectional and multidirectional random waves obtaining a good agreement with the laboratory data.

The previous theories give reasonable results in the majority of waves of maritime interest. The open question is the definition of ranges in which the best method should be selected. A contribution in answering to this question was given by *Hattory (1986)* who suggested a method to recognise the best fitting theory by comparing the maxima and minima theoretical and experimental velocities. *Vis (1980)* assumed the error to be equal to the mean of the difference

between the theoretical and measured spectral frequency components rated by the latter one.

The experiments hereafter presented have been used to analyse in frequency domain the validity of some of the previous theories for regular waves, and the criteria suggested by Vis have been used in evaluating the errors.

The velocity field was measured using a LDA system (Damiani and Mossa, 1996a). A 5 W Ar-Ion laser was used; in the transmitter, the incoming beam from the laser is divided into two pairs of laser beams of green and blue colors for measurements of two velocity components. At the same time a frequency shift is added to one beam of each beam pair to allow the measurements of the reversing flow.

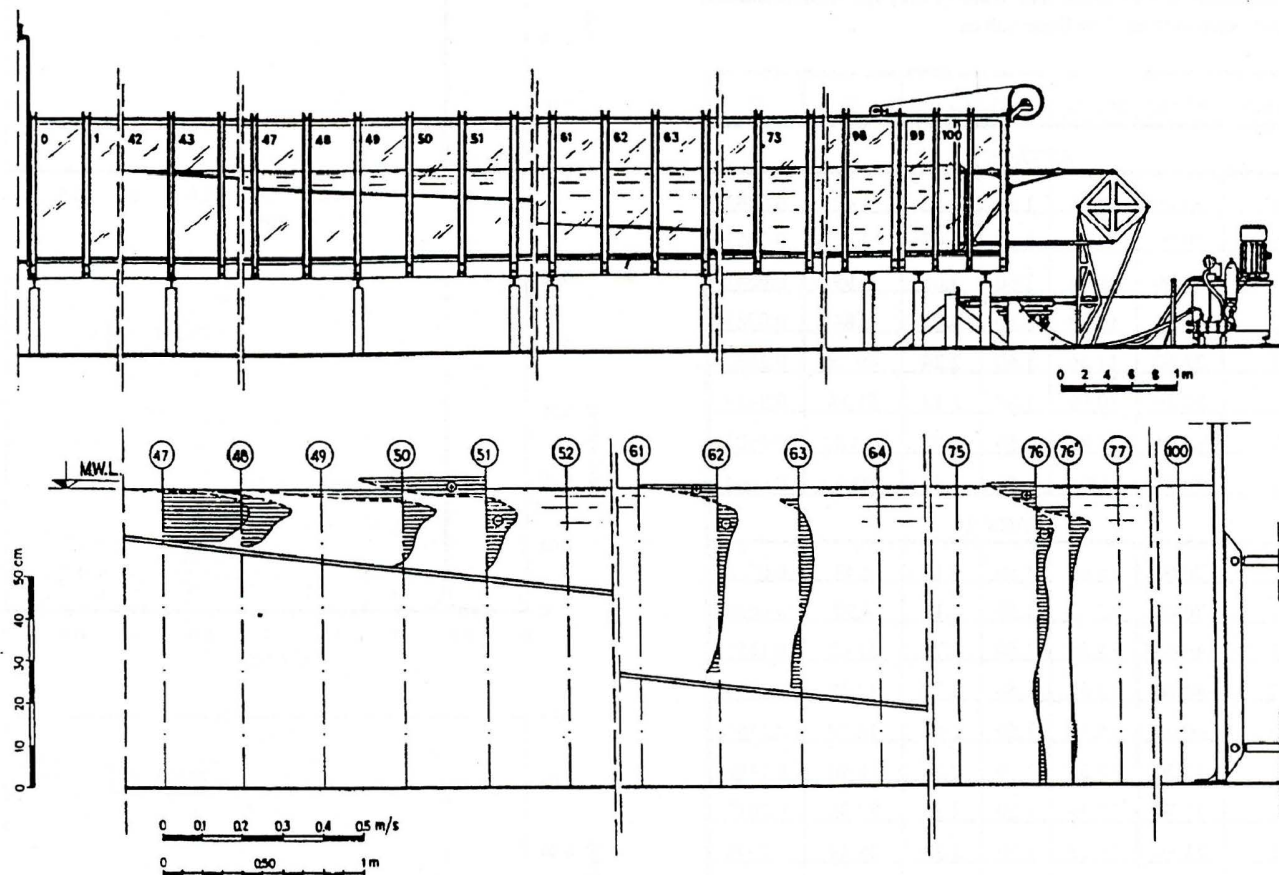


Fig. 1 Wave channel and horizontal mean velocity profiles.

EXPERIMENTAL SET-UP

The experiments were carried out in the wave flume at the laboratory of the Water Engineering Department of the Polytechnic of Bari (figure 1). The channel is about 45 m long and 1 m wide; its walls consist of crystal glass sheets 1.2 m high, supported by iron frames with a center to center distance of about 0.44 m, where wave measurement sections (numbered from the shoreline to the wave paddle) have been located. The channel has a 1/20 sloped wooden bottom from the shoreline to section 73 and a horizontal bottom from section 73 to the wave paddle; during the tests the mean water depth in the channel near the paddle was 0.7 m. The wave generation system consists of a flat paddle which receives a rotatory-translational motion, through a kinematic mechanism with an oleodynamic system driven by an electrical valve and controlled by a process computer.

A probe with optic fibre cable is connected to the transmitter using precision-adjustable manipulators. A Dantec LDA signal processor (58N40 FVA Enhanced), based on covariance techniques, was used. Wave elevations were measured by means of resistance probes simultaneously to velocity acquisition. This allows the phase shifting between elevations and velocity components to be evaluated.

In table 1 the main characteristics of the tested waves are reported in the sections where measurements of both elevations and velocity components were made. L_A represents the wave length evaluated with Airy theory. The last two columns report the Ursell number and Goda non-linearities parameter respectively.

Figure 1 shows the experimental mean horizontal velocity for the first attack of table 1, together with the experimental channel. The plus sign (the left side of the diagrams with respect to the measurement section) indicates an onshore velocity direction, while the minus sign (the right side of the diagrams) shows an offshore mean velocity direction. It can be observed that in the sections nearest the shoreline a strong offshore directed current is present.

This is due to the breaking occurring around section 48, thus, the undertow current takes place and turbulence is the dominant action governing the process. This region will be disregarded in the discussion of results because of the poor consistency of the theoretical hypotheses.

In many cases the velocity components between trough and crest were measured. Unlike *Steve and Wind (1982)* the instrumentation used here permits to analyse these values.

| SECTION | h [cm] | H [cm] | T [s] | L _A [m] | U | Π |
|----------|--------|--------|-------|--------------------|--------|--------|
| ATTACK 1 | | | | | | |
| 76' | 70.00 | 9.97 | 1.60 | 3.43 | 3.42 | 0.0462 |
| 76 | 70.00 | 9.82 | 1.60 | 3.43 | 3.37 | 0.0455 |
| 63 | 48.80 | 8.73 | 1.60 | 3.05 | 6.99 | 0.0643 |
| 62 | 46.80 | 10.00 | 1.60 | 3.01 | 8.84 | 0.0783 |
| 51 | 22.60 | 11.36 | 1.60 | 2.24 | 49.38 | 0.2880 |
| 50 | 20.10 | 10.95 | 1.60 | 2.13 | 61.18 | 0.3419 |
| 48 | 16.01 | 13.85 | 1.60 | 1.92 | 124.42 | 0.6502 |
| 47 | 10.75 | 7.30 | 1.45 | 1.44 | 121.85 | 0.6064 |
| ATTACK 2 | | | | | | |
| 76' | 70.00 | 15.60 | 1.50 | 3.12 | 4.43 | 0.0716 |
| 76 | 70.00 | 15.03 | 1.50 | 3.12 | 4.27 | 0.0690 |
| 63 | 46.60 | 15.50 | 1.50 | 2.76 | 11.67 | 0.1158 |
| 62 | 44.60 | 14.61 | 1.50 | 2.72 | 12.18 | 0.1159 |
| 60 | 40.20 | 14.19 | 1.50 | 2.62 | 14.99 | 0.1305 |
| 58 | 35.50 | 13.53 | 1.50 | 2.50 | 18.90 | 0.1498 |
| 56 | 31.70 | 15.26 | 1.50 | 2.39 | 27.36 | 0.2012 |
| 54 | 26.90 | 15.32 | 1.50 | 2.24 | 39.49 | 0.2638 |
| 52 | 22.20 | 15.32 | 1.50 | 2.07 | 60.00 | 0.3654 |
| 50 | 17.80 | 16.71 | 1.50 | 1.87 | 103.61 | 0.5820 |
| 48 | 13.80 | 8.38 | 1.43 | 1.59 | 80.61 | 0.4298 |

Table 1

RESULTS AND DISCUSSION

Figures 2 and 3 show the amplitude spectra of wave elevations, horizontal and vertical velocity components, obtained from measurements in a point of section 76 and 51. The velocities were gauged at a distance of 32 and 16 cm from the bottom respectively. It can be seen that in the offshore section the wave spectra present only a harmonic of fundamental frequency, with a typical 2nd order Stokes behaviour.

In figure 2, also the results of the 2nd order Stokes theory are shown. It has to be pointed out that the methods hereafter used to evaluate velocity components start from the knowledge of the real wave profile assessed in the channel. On the contrary, Stokes theory gives joint theoretical results for wave elevations and velocity

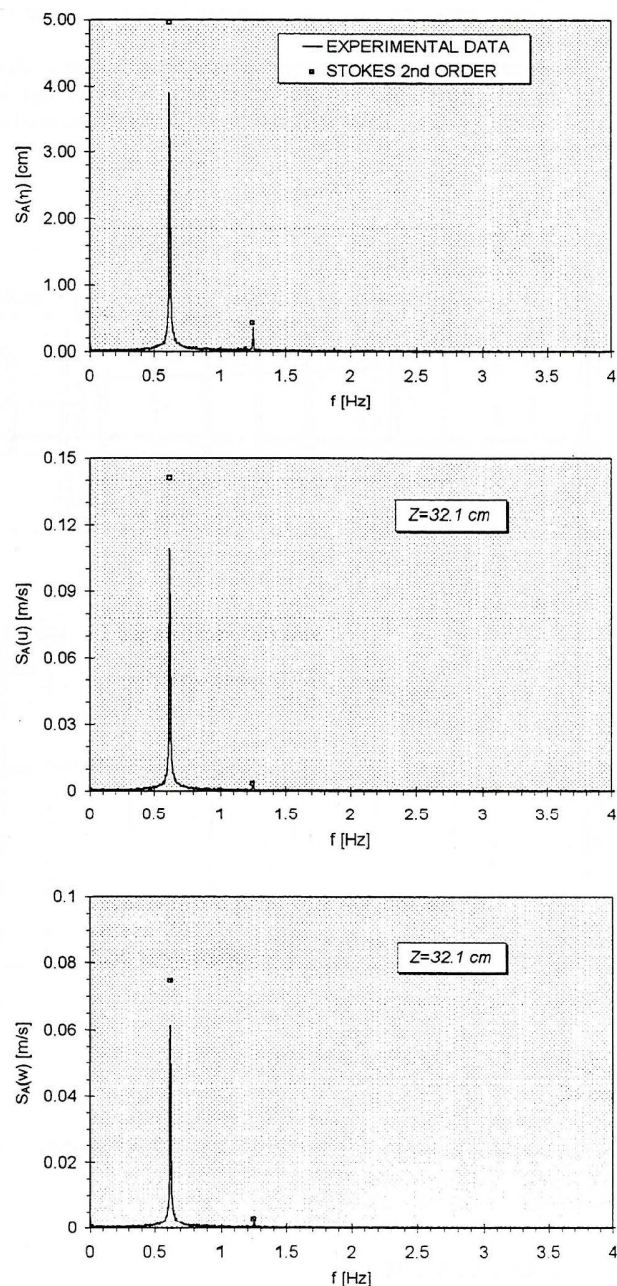


Fig. 2 Amplitude spectra of elevations, horizontal and vertical velocity components assessed in section 76.

components; this makes the comparison between Stokes results and those of all other methods here used difficult. For this reason in the following pages the results of 2nd and 3rd order Stokes calculations will not be presented, also because it has been verified that in all the examined situations they are worse than those obtained with the methods assuming the real elevations as an input.

When the wave goes up to the breaking region, other harmonics appear, clearly showing that the shoaling produces a deformation of the waves and the consequent increase of non-linearities.

Figure 3 shows that the vertical velocity component is more affected by non-linearities, indeed the rate between higher harmonic

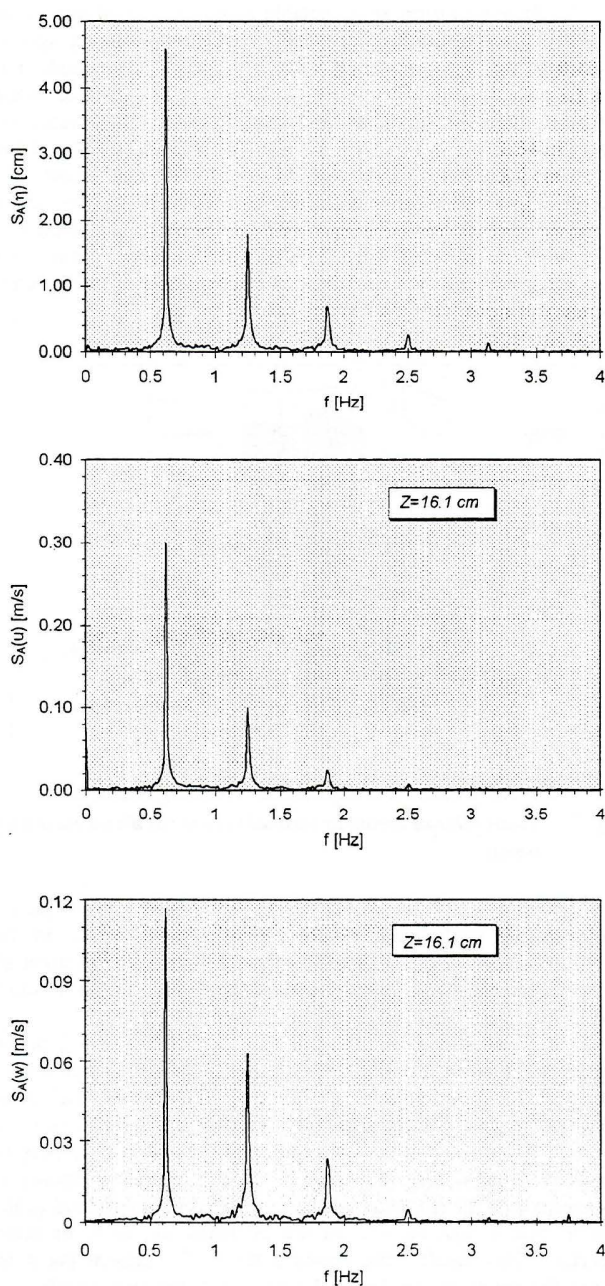


Fig. 3 Amplitude spectra of elevations, horizontal and vertical velocity components assessed in section 51.

and fundamental peak spectral density values is greater in vertical amplitude spectra than in the horizontal ones (Damiani and Mossa, 1995a).

The following analysis of velocity spectra has been limited to the first two peaks for homogeneity reasons, because the examined spectra in the deeper water sections have only two peaks as shown in figure 2.

Figures 4 and 5 show the first two peaks of previous velocity component spectra, together with the values coming from transfer function methods. For clarity reasons the two frequency peaks have been reported with different scales.

Chakrabarti's approach (Chakrabarti, 1971) and its 2nd order approximation (Gudmestad and Connor, 1986), respectively CH I and CH II, have been applied using time history elevation as an input. In the same way, the methods reported by Koyama and Iwata (1986) have been applied using equations (1) in which elevations are known. The transfer function has a linear form (Linear function in Time Domain) or the modified form suggested by the Authors (Modified method in Time Domain) and it assumes anyhow only one value for a fixed measurement point. Although the results are presented in spectral form, the previous methods propose a linkage between time series of elevations and velocity components, that is the water surface profile is not decomposed into Fourier components. Thus the adopted procedure consists of calculating the time history of velocity components firstly and then of assessing the relative

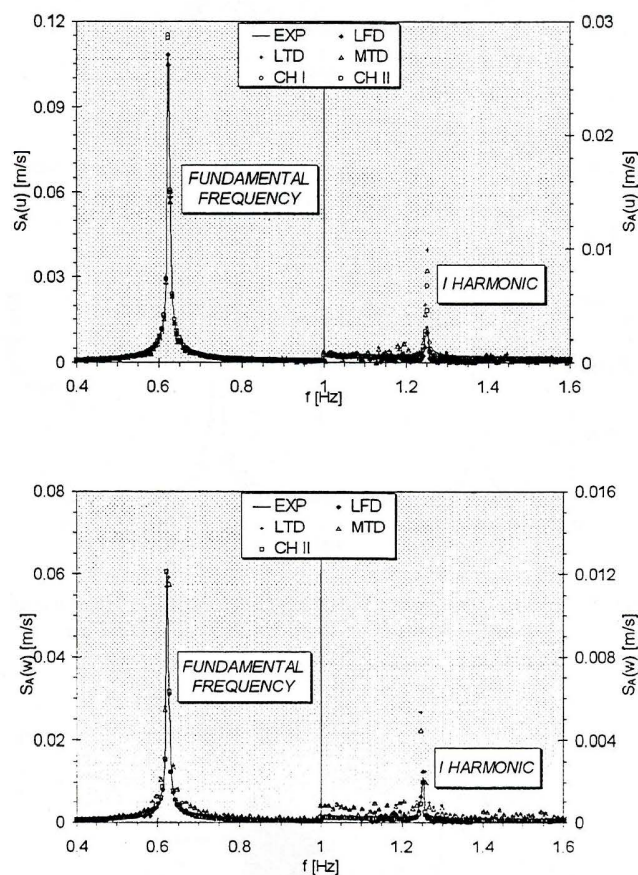


Fig. 4 Experimental and theoretical amplitude spectra of u and w components assessed in section 76 at $Z=32.1$ cm.

amplitude spectra by using FFT algorithm. In applying Koyama and Iwata methods, the value of Δt in equation (1) is the experimental time lag from a zero-upcrossing point to the next coming wave crest.

Finally, a linear transfer function method was applied to directly evaluate the velocity amplitude spectra in frequency domain, starting from wave elevation ones (Linear method in Frequency Domain), as suggested by Woltering and Daemrich (1995). In this case the method was applied only to evaluate the fundamental peak and its harmonics by extending the computation to a narrow frequency band around each peak ($0.9 f \div 1.1 f$). In this case the transfer function

assumes a different value for each Fourier component of the wave elevations and the phase shift between w and η is $\pi/2$.

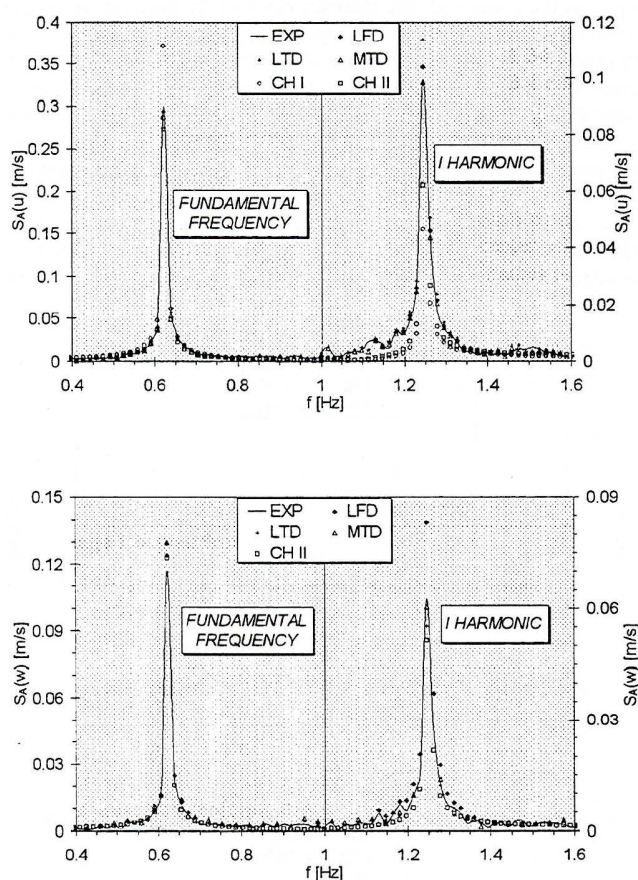


Fig. 5 Experimental and theoretical amplitude spectra of u and w components assessed in section 51 at $Z=16.1$ cm.

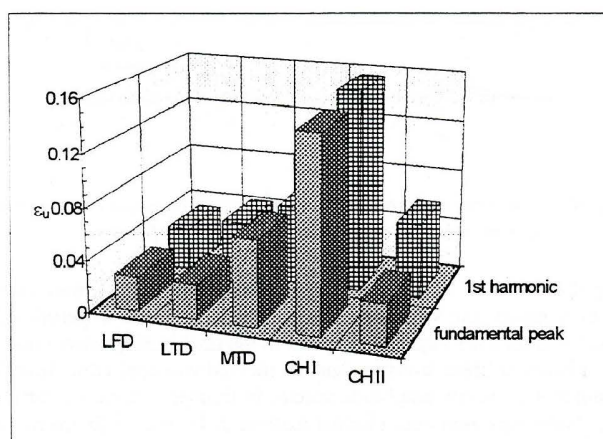


Fig. 6 Mean relative errors of horizontal velocity at all measurement points.

The figures confirm an acceptable approximation of almost all methods. The analysis was repeated for all measurement points to investigate the errors occurring when using different methods. For each frequency component the relative error can be evaluated as the difference between predicted and experimental values rated by experimental ones; in this way, the sign of the error indicates if the method overpredicts or underpredicts the experimental values. The average of previous absolute errors can be assumed as a quality index for the different methods. This procedure is the same one adopted by *Vis (1980)*; in this case only the first two frequency peaks were considered, whereas *Vis* calculated the mean error of all spectral frequency components for analysing random waves.

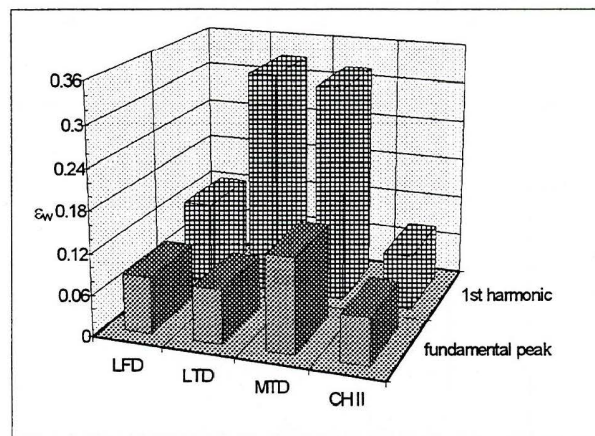


Fig. 7 Mean relative errors of vertical velocity at all measurement points.

Figures 6 and 7 report the errors on the first two frequency peaks obtained as the mean values of the absolute errors on all the spectra measured in the channel independently from the position of the measurement point (section and distance from the still water level).

First of all, it can be observed that all methods give a better approximation when predicting the horizontal velocity component u . Moreover, for both velocity components, the fundamental peak evaluation is generally more accurate than the 1st harmonic one. It has to be mentioned that the contribution of the fundamental peak is usually more important when determining the velocity components; this means that the influence of the error on the fundamental peak, even if smaller than the one on the 1st harmonic, could be more important. This justifies the choice made in the present paper to neglect the errors on the 2nd and following harmonics, usually too small with respect to the fundamental peak. On this subject it has to be underlined that *Vis (1980)*, analysing random waves, neglected all spectral density values smaller than 10% of peak one, considering them quite inaccurate.

In figure 8, in the same way with *Vis (1980)*, the previously discussed mean values between fundamental frequency peak and 1st harmonic errors are shown both for horizontal and vertical velocity components. It can be stated that the method which better approximates the experimental data is the linear transfer function applied in frequency domain concerning the horizontal velocity component, while for the vertical one Chakrabarti 2nd order is preferable; this confirms that not always the same method gives best results for all wave kinematics (*Dean, 1970*).

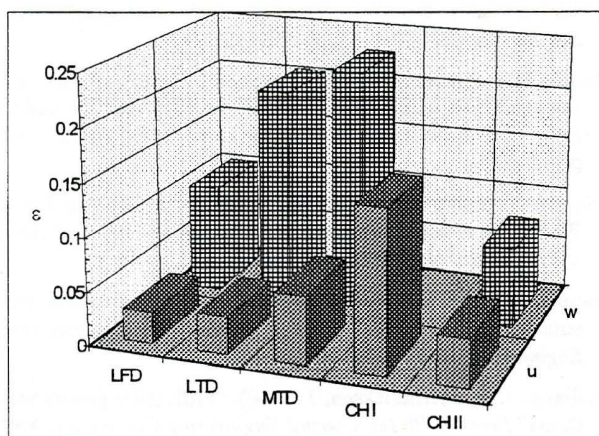


Fig. 8 Mean errors of velocity components obtained by averaging the fundamental and 1st harmonic peak errors.

In figures 9 and 10, the depth averaged errors for each method are shown against the sections where measurements have been made. Each bar represents the mean error in a section where many measurement points located at different distances from the bottom were investigated. The figure confirms that the u component can be assessed with a better approximation than the vertical one everywhere in the channel.

A tendency of all methods to worsen the horizontal velocity evaluation when approaching the breaking region can be noted. This consideration is evident for CH I which is not suitable in evaluating non-linearities. Chakrabarti's 2nd order approximation (CH II) was successfully introduced to refine the CH I.

About the LFD method it has to be noted that in spite of its simplicity and non rigorous assumption (it is not obvious that linear theory will adequately relate u and η for clearly non-linear waves), the results obtained are pretty well approximated.

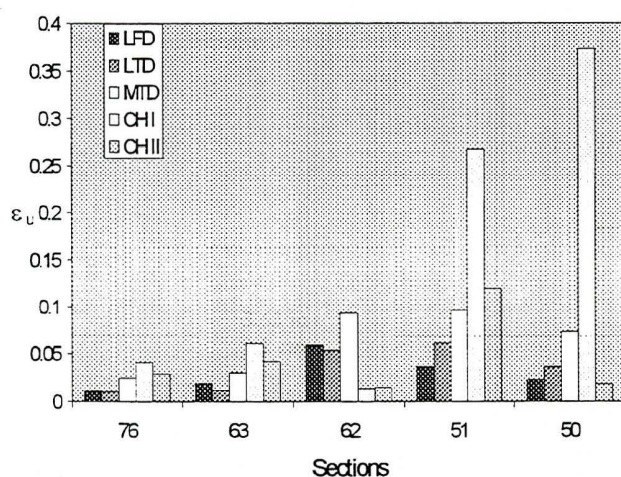


Fig. 9 Mean errors in horizontal velocity predicting against the measurement sections.

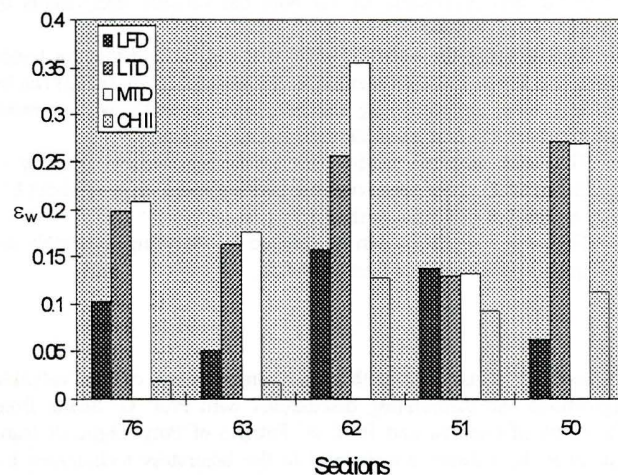


Fig. 10 Mean errors in vertical velocity predicting against the measurement sections.

CONCLUSIONS

In the present paper the velocity components of regular waves have been analysed by comparing theoretical values with measured ones. The theoretical values of velocity components have been obtained applying five approximate methods which predict the wave orbital velocities from wave elevations.

The five methods considered are Chakrabarti's approach (Chakrabarti, 1971) and its 2nd order approximation (Gudmestad and Connor, 1986), the two transfer function methods in time domain reported by Koyama and Iwata (1986) and a linear transfer function method in frequency domain.

The comparison between calculated and measured velocity amplitude spectra shows that all methods generally predict the motion field with an acceptable approximation. The CH I method is the only one that shows remarkable errors in the computing of horizontal component in those sections where the non-linearities are more visible. In this case the method has to be substituted with Chakrabarti's 2nd order approximation. It is also stated that all methods give a more reliable approximation of the fundamental peak than the first harmonic.

Moreover the predicted horizontal components are more accurate than the vertical ones. This is due to the fact that the vertical components are more strongly affected by non-linearities. The previous results have been depth averaged, since many measurement points have been analysed in each section from the bottom to the wave profile level. It has to be mentioned that no evident dependence has been found between the errors and the quantity Z/h .

The present investigation has also shown that a method giving the best approximation everywhere in the channel does not exist, and that it should be investigated the range of validity of each method. For practical proposals, LFD and CH II methods can be generally used better than the other ones.

Moreover it has to be observed that it is not always possible to obtain the best representation for both the velocity components by using the same method.

By averaging the obtained results, it can be said that for tested waves, the linear function method in frequency domain (LFD) can be generally used in predicting the horizontal velocity, with a mean error of about 4% and a maximum one smaller than 10%.

The most suitable method for predicting vertical velocity is Chakrabarti 2nd order approximation, with a mean error of about 6% and a maximum one smaller than 15%.

The same conclusion can be reached by analysing separately the fundamental peak and the first harmonic.

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REFERENCES

- Chakrabarti, SK (1971). Discussion on "Dynamics of single point mooring in deep water" *Journal of Waterway, Harbour and Coastal Engr. Division*, ASCE, 97 (WW3), pp 588-590.
- CERC (1994). "Shore protection manual" *Department of the Army, US Army Corps of Engineers*, Washington, DC 20134.
- Damiani, L, and Mossa, M (1996). "Indagine sul campo di moto di un'onda regolare" *Scritti in Onore di Mario Ippolito, Ass. Idrotecnica Italiana Sez. Campana - Dip. di Ing. Idraulica ed Ambientale Girolamo Ippolito*, Napoli, pp 227-238.
- Damiani, L, and Mossa, M (1996). "Ricostruzione di un campo di moto ondoso" *Accademia Pugliese delle Scienze*, Bari - Italy.
- Dean, RG (1970). "Relative validities of water wave theories" *J. Waterway and Harbors Div., Proc. of ASCE*, Vol 96, No WW1, pp 105-119.
- Fenton, JD (1985). "A fifth-order Stokes theory for steady waves" *J. Waterway Port Coastal and Ocean Eng.*, ASCE, Vol 111, No 2, pp 216-234.
- Graw, KU (1994). "Comparison of wave theories with velocity measurement" *Proc. of Int. Symp. Waves-Physical and Numerical Modelling*, University of British Columbia, Vancouver, pp 561-569.
- Gudmestad, OT, and Connor, JJ (1986). "Engineering approximation to nonlinear deepwater waves" *J. of Applied Ocean Research*, Vol 8, No 2, pp 76-88.
- Guza, RT, and Thornton, EB (1980). "Local and shoaled comparisons of sea surface elevations, pressures, and velocities" *J. of Geophysical Research*, Vol 85, No C3, pp 1524-1530.
- Hattori, M (1986). "Experimental study on the validity range of various wave theories" *Proc. of 20th Int. Conf. Coastal Engineering*, ASCE, Vol 1, pp 232-246.
- Koyama, H, and Iwata, K (1986). "Estimation of particle velocities of shallow water waves by a modified transfer function method" *Proc. of 20th Int. Conf. on Coastal Engineering*, ASCE, Vol 1, pp 425-436.
- Rebaudengo Landò, L, and Scarsi, G (1995). "Directional Random Wave Kinematics: Third Order Approximation" *Proc. of 5th Int. Offshore and Polar Eng Conference*, Vol III, pp 49-56.
- Sawaragi, T (1995). "Coastal engineering-waves, beaches, wave-structure interactions" *Developments in Geotechnical Engineering*, 78, Ed. Elsevier.
- Skjelbreia, L, and Hendrickson, J (1960). "Fifth order gravity waves theory" *Proc. of 7th Int. Coastal Engineering Conference*, ASCE, pp 184-196.
- Steve, MJF, and Wind, HG (1982). "A study of radiation stress and set-up in the nearshore region" *Delft Hydraulics*, Publication No 267.
- Ting, FCK, and Kirby, JT (1995). "Dynamics of surf-zone turbulence in a strong plunging breaker" *J. of Coastal Engineering*, Vol 24, pp 177-204.
- Vis, FC (1980). "Orbital velocities in irregular waves" *Delft Hydraulics*, Publication No 231.
- Woltering, S, and Daemrich, KF (1994). "Regular wave investigations of wave kinematics with lagrangian approach" *Proc. of International Symposium: Waves-Physical and Numerical Modelling*, Vancouver, pp 665-673.
- Woltering, S, and Daemrich KF (1995). "Experimental validity of wave theories and application to horizontal orbital velocities" *International Conference on Coastal and Port Engineering in Developing Countries*, Rio de Janeiro, Vol 3, pp 2411-2424.